Motion Estimation

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Presentation Outline

How do you get keypoint correspondence?

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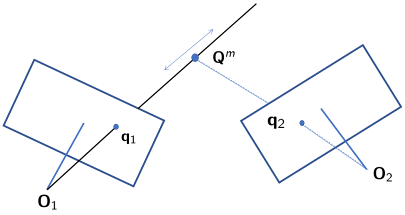
- We use keypoint and descriptor matching algorithms, e.g., SIFT, BRIEF, etc.
- What kind of constraints exist on the point correspondences in two images?
  - Epipolar constraint

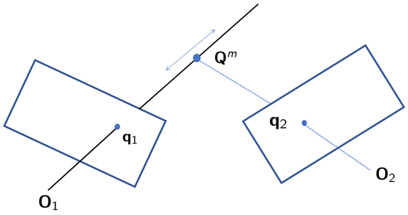
Presentation Outline

What can you say about matching pixels?



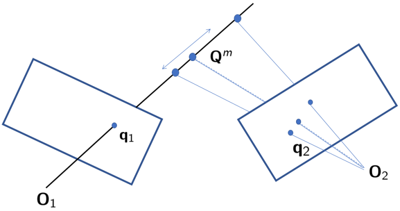
- Assume that we are given the calibration, rotation, and translation parameters for the two cameras.
- We are given a single pixel $\mathbf{q}_1$ in the left image.
- Let $\mathbf{q}_2$ be the unknown pixel in the second image corresponding to $\mathbf{q}_1$.
- Given $\mathbf{q}_1$ can we find the location of $\mathbf{q}_2$?
  - NO!

What can you say about matching pixels?



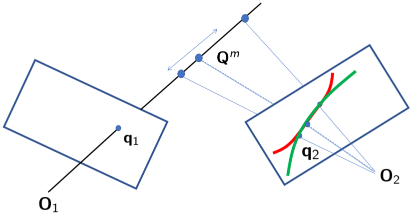
- For simplicity, we don't show the optical axis.

What can you say about matching pixels?



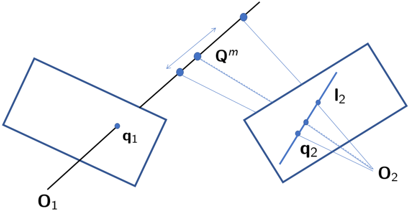
- We consider different 3D points $\mathbf{Q}^m$ on the backprojection of $\mathbf{q}_1$.
- We look at the forward projections of these 3D points on the right image.
- The different projections are the different possibilities for $\mathbf{q}_2$ given the position of $\mathbf{q}_1$.

What can you say about matching pixels?



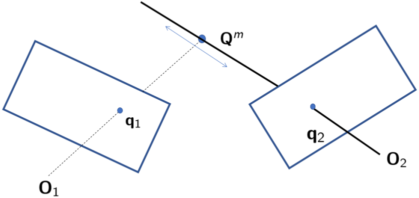
- What is the parametric curve that passes through different possible locations of $\mathbf{q}_2$?

What can you say about matching pixels?



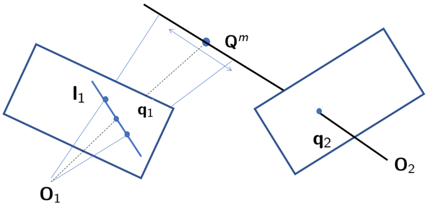
- It is a straight line.

What can you say about matching pixels?



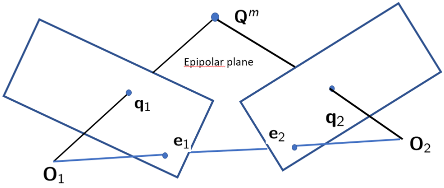
- What can you say if $\mathbf{q}_2$ is given and we are interested in finding the location of $\mathbf{q}_1$.

What can you say about matching pixels?



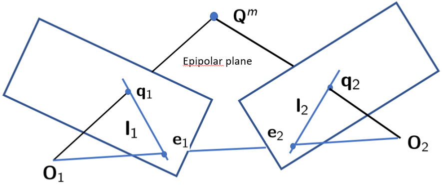
- Yes, it is also a straight line.
- Given a pixel in one image, the corresponding pixel in the other image is constrained to lie on a straight line.

Epipolar Plane and Epipoles



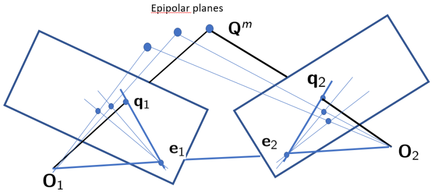
- **Epipolar plane** is the plane formed by the two camera centers $(\mathbf{O}_1, \mathbf{O}_2)$ and a 3D point $\mathbf{Q}^m$.
- The line joining the two camera centers intersect the image planes at points that we refer to as **epipoles**.
- The epipole in the first image is denoted by $\mathbf{e}_1$. The epipole in the second image is denoted by $\mathbf{e}_2$.

Epipolar Lines



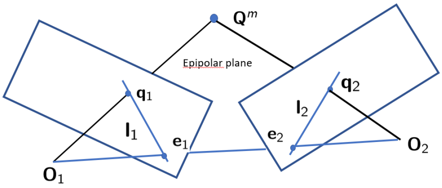
- Given a pixel $\mathbf{q}_1$, the corresponding pixel $\mathbf{q}_2$ lies on a line in the right image that we refer to as epipolar line $\mathbf{l}_2$. Note that this line passes through the epipole $\mathbf{e}_2$.
- The epipolar line in the first image is denoted by $\mathbf{l}_1$ and it joins $\mathbf{q}_1$ and $\mathbf{e}_1$.
- Note that the epipoles depend only on rotation, translation, and calibration parameters of the two cameras.

Family of epipolar planes



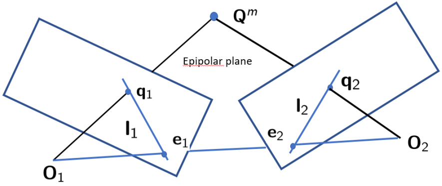
- For every pair of matching pixels, we can think of an epipolar plane formed by the optical centers and the 3D point.
- All the epipolar planes pass through the epipoles. Thus the epipolar lines can be seen as family of lines passing through a single point.

Derivation of the epipolar line



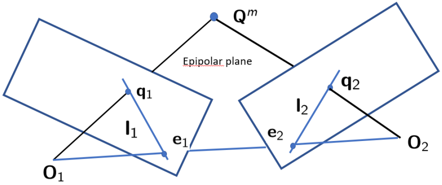
- Given a pixel $\mathbf{q}_1$, the corresponding pixel $\mathbf{q}_2$ lies on epipolar line $\mathbf{l}_2$.
- The epipolar line $\mathbf{l}_2$ in the right image is the line joining the $\mathbf{e}_2$ and $\mathbf{q}_2$ on the right image.
- Let the forward projections be given by: $\mathbf{q}_1 \sim K_1 R_1 (I \mid -\mathbf{t}_1)\mathbf{Q}^m$. $\mathbf{q}_2 \sim K_2 R_2 (I \mid -\mathbf{t}_2)\mathbf{Q}^m$.

Derivation of the epipolar line



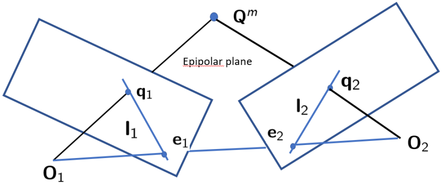
- The epipole $\mathbf{e}_2$ is the projection of the left camera center on the right image. The left camera center is given by $\mathbf{t}_1$.
- A 3D point on the back-projected ray of $\mathbf{q}_1$ is given by $\lambda_1 R_1^T K_1^{-1} \mathbf{q}_1 + \mathbf{t}_1$. We obtain $\mathbf{q}_2$ by projecting this point on the right image.

Derivation of the epipolar line



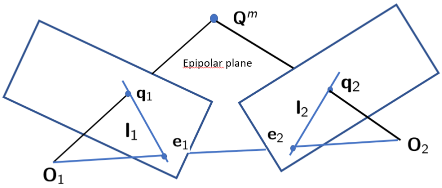
- $$\mathbf{e}_2 \sim K_2 R_2 (I \mid -\mathbf{t}_2) \begin{pmatrix} \mathbf{t}_1 \\ 1 \end{pmatrix}$$
  $$\mathbf{q}_2 \sim K_2 R_2 (I \mid -\mathbf{t}_2) \begin{pmatrix} \lambda_1 R_1^T K_1^{-1} \mathbf{q}_1 + \mathbf{t}_1 \\ 1 \end{pmatrix}$$

Derivation of the epipolar line



- $$\mathbf{e}_2 \sim \mathrm{K}_2\mathrm{R}_2(\mathbf{t}_1 - \mathbf{t}_2)$$
  $$\mathbf{q}_2 \sim \mathrm{K}_2\mathrm{R}_2(\lambda_1 \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1 + (\mathbf{t}_1 - \mathbf{t}_2))$$

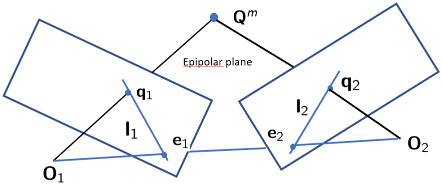
Derivation of the epipolar line



- The epipolar line $\mathbf{l}_2$ can by obtained from the cross-product of $\mathbf{e}_2$ and $\mathbf{q}_2$.
- Note that $\mathrm{M}\mathbf{x} \times \mathrm{M}\mathbf{y} \sim \mathrm{M}^{-T}(\mathbf{x} \times \mathbf{y})$.
- Thus we have:

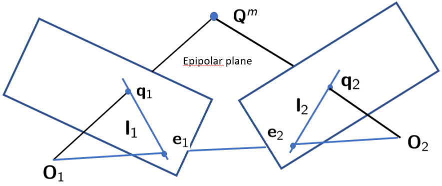
$$\begin{aligned}\mathbf{l}_2 &\sim \mathbf{e}_2 \times \mathbf{q}_2 \\ &\sim \mathrm{K}_2\mathrm{R}_2(\mathbf{t}_1 - \mathbf{t}_2) \times \mathrm{K}_2\mathrm{R}_2(\lambda_1 \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1 + (\mathbf{t}_1 - \mathbf{t}_2))\end{aligned}$$

Derivation of the epipolar line



- $$\mathbf{e}_2 \times \mathbf{q}_2 \sim (\mathrm{K}_2\mathrm{R}_2)^{-T}((\mathbf{t}_1 - \mathbf{t}_2) \times (\lambda_1 \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1 + (\mathbf{t}_1 - \mathbf{t}_2))$$
- Since $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ and $\mathbf{a} \times \mathbf{a} = \mathbf{0}$, we have:
  $$\mathbf{l}_2 \sim (\mathrm{K}_2\mathrm{R}_2)^{-T}((\mathbf{t}_1 - \mathbf{t}_2) \times \lambda_1 \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1)$$
  We can remove λ_1 since the relation is up to a scale:
  $$\mathbf{l}_2 \sim (\mathrm{K}_2\mathrm{R}_2)^{-T}((\mathbf{t}_1 - \mathbf{t}_2) \times \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1)$$

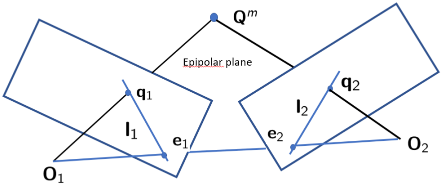
Derivation of the epipolar line



- $$\mathbf{l}_2 \sim (\mathrm{K}_2\mathrm{R}_2)^{-T}((\mathbf{t}_1 - \mathbf{t}_2) \times \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1)$$
- Skew-symmetrix matrix of any 3×1 vector $\mathbf{a}$ is given below:

$$[\mathbf{a}]_\times = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

Derivation of the epipolar line

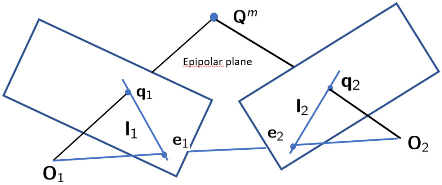


- $$\mathbf{l}_2 \sim (\mathrm{K}_2\mathrm{R}_2)^{-T}((\mathbf{t}_1 - \mathbf{t}_2) \times \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1)$$
- We know that the cross-product of two 3×1 vectors $\mathbf{a}$ and $\mathbf{b}$ can be written as follows:

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b}$$

- $$\mathbf{l}_2 \sim (\mathrm{K}_2\mathrm{R}_2)^{-T}([\mathbf{t}_1 - \mathbf{t}_2]_\times \mathrm{R}_1^T \mathrm{K}_1^{-1}\mathbf{q}_1)$$

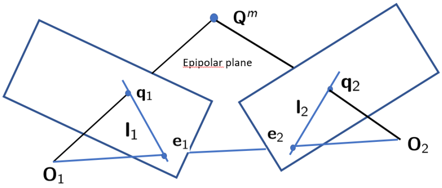
Derivation of the epipolar line



- $$\mathbf{l}_2 \sim (K_2R_2)^{-T}([\mathbf{t}_1 - \mathbf{t}_2]_\times R_1^T K_1^{-1}\mathbf{q}_1)$$
  $$\mathbf{l}_2 \sim (K_2R_2)^{-T}[\mathbf{t}_1 - \mathbf{t}_2]_\times (R_1^T K_1^{-1})\mathbf{q}_1$$
- Here we can see the transformation of a point $\mathbf{q}_1$ in the left image to a line $\mathbf{l}_2$ in the right image using a 3×3 matrix $(K_2R_2)^{-T}[\mathbf{t}_1 - \mathbf{t}_2]_\times(R_1^T K_1^{-1})$.

Presentation Outline

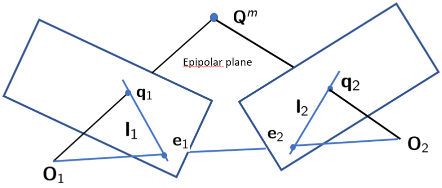
Fundamental Matrix



- The 3×3 matrix is the celebrated fundamental matrix: $F_{12} = (K_2R_2)^{-T}[\mathbf{t}_1 - \mathbf{t}_2]_\times(R_1^TK_1^{-1})$
- This matrix encodes the epipolar geometry.
- We know that $\mathbf{q}_2^T\mathbf{l}_2 = 0$. Thus we have the following:

$$\mathbf{q}_2^TF_{12}\mathbf{q}_1 = 0$$

Fundamental Matrix



- We can have the following equation based on the epipolar line $\mathbf{l}_1$

$$\mathbf{q}_1^T F_{21} \mathbf{q}_2 = 0$$

- For simplicity we will only consider the following equation:

$$\mathbf{q}_2^T F \mathbf{q}_1 = 0$$

- This constraint is the so-called **epipolar constraint**.

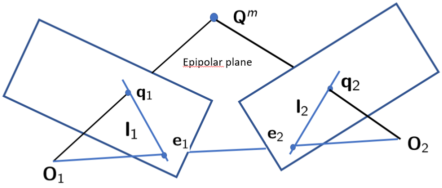
Computation of Fundamental matrix

- Calibration matrices:

$$K_1 = K_2 = \begin{pmatrix} 200 & 0 & 320 \\ 0 & 200 & 240 \\ 0 & 0 & 1 \end{pmatrix}$$

- Rotation matrices:$R_1 = R_2 = I$.
- Translation matrices: $\mathbf{t}_1 = \mathbf{0}, \mathbf{t}_2 = (100, 0, 0)^T$.
- Correspondences: $\mathbf{q_1} = (520, 440, 1)^T, \mathbf{q_2} = (500, 440, 1)^T$
- Compute the fundamental matrix F and show that $\mathbf{q}_2^T F \mathbf{q}_1 = 0$.
- Find the two epipoles and epipolar lines.

Computation of the fundamental matrix



- Epipolar constraint:$\mathbf{q}_2^T \mathrm{F} \mathbf{q}_1 = 0$
- Using n point correspondences we can rewrite the above equation of the following form:

$$\mathrm{A}\mathbf{f} = 0$$

Here **A** is a $n \times 9$ matrix consisting of only the coordinates of the point correspondences that are known. The 9×1 vector f consists of 9 unknowns from the 3×3 fundamental matrix F.

Computation of the fundamental matrix

- Using n point correspondences, we can have the following equation:

$$A_{n \times 9}\mathbf{f} = 0$$

$$A = \begin{pmatrix} q_{1x}q_{2x} & q_{1y}q_{2x} & q_{2x} & q_{1x}q_{2y} & q_{1y}q_{2y} & q_{2y} & q_{2x} & q_{2y} & 1 \\ & . & . & . & . & . & . & . & . \\ & . & . & . & . & . & . & . & . \end{pmatrix}$$

$$f = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{21} & f_{22} & f_{23} & f_{31} & f_{32} & f_{33} \end{pmatrix}^T$$

- Show the $n \times 9$ matrix using the point correspondences $\{\mathbf{q}_1, \mathbf{q}_2\} = \{(u_{1i}, v_{1i}), (u_{2i}, v_{2i})\}, i = \{1 \cdots n\}$.

- To find the solution of the equation $A\mathbf{f} = \mathbf{0}$, we first compute SVD of A, i.e.,$[U, S, V] = SVD(A)$ and then the solution of f is given by the last column of V.
- The rank of A should be 8 if we use 8 point correspondences.

Acknowledgments

Some presentation slides are adapted from the following materials:

- Peter Sturm, Some lecture notes on geometric computer vision (available online).