CS 6320 Computer Vision

Homework 4 (Due Date – April 1st)

This assignment is not a programming assignment, but the general framework of Boolean and multi-label (alpha-expansion, alpha-beta swap) can be applied to many computer vision problems such as segmentation and stereo.

1. Show that the following function (all the variables are Boolean) is submodular using the set definition (i.e., $f(A) + f(B) \geq f(A \cup B) + f(A \cap B)$):

   $f(x_1, x_2, x_3) = 2x_1 - x_1x_2 - 2x_2x_3$

   Note that for every pseudo-Boolean function, you can think of an associated set function. In order to show that the above function is submodular, you will need to show that the equation holds for all possible subsets A and B. For the assignment, it is sufficient if you show the results for only 6 possible subsets. [10 points]

2. Show that the following function (all the variables are Boolean) is not submodular using the set definition (i.e., $f(A) + f(B) \geq f(A \cup B) + f(A \cap B)$):

   $$f(x_1, x_2, x_3) = 2x_1 - x_1x_2 + 2x_2x_3$$

   To show that a given function is not submodular, you will need to show one counterexample. Find some subsets A and B for which the above function does not satisfy submodularity.

   [10 points]

3. Show the maxflow/mincut graph for the following equation:

   $f(x_1, x_2, x_3, x_4) = -2x_1 + 3x_2 + 5x_3 + 7x_4 - x_1x_2 - 2x_2x_3 - 4x_1x_4 - 5x_2x_4$

   Manually identify the **best** solution (all the variables are Boolean) and show the corresponding cut on the graph. In other words, you will consider all possible states for the Boolean variables and identify the solution that has the minimum value. Show that the cost of the cut matches with the cost of the function (without the introduced constant term).

   [10 points]

4. Show the pseudo-Boolean function associated with the following maxflow/mincut graph shown in Fig. 1.

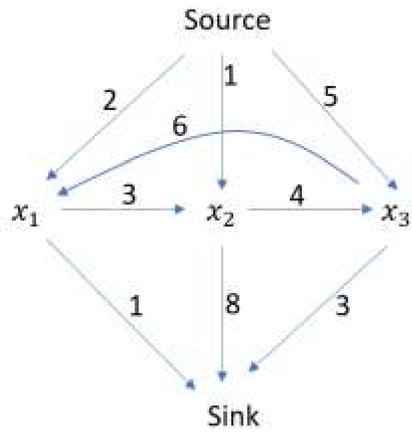
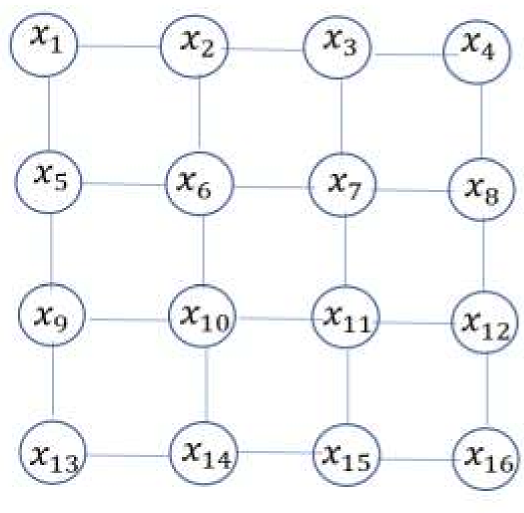


Figure 1: There are two terminal nodes (source denoting class 0, and sink denoting class 1). There are three non-terminal nodes.

Manually identify the **best** solution (all the variables are Boolean) and show the corresponding cut on the graph. In other words, you will consider all possible states for the Boolean variables and identify the solution that has the minimum value. Show that the cost of the cut matches with the cost of the function (without the introduced constant term). [10 points]

5. Consider the following image of size 4x4 as shown in Fig. 2.



(a)

[2,1]	[2,1]	[2,1]	[2,1]
[2,1]	[1,2]	[1,2]	[2,1]
[2,1]	[1,2]	[1,2]	[2,1]
[2,1]	[2,1]	[2,1]	[2,1]

(b)

Figure 2: (a) We show a 4x4 image with 16 pixels. We use 4-connected neighborhood for adjacent pixels. (b) Unary costs are shown for every pixel in the form [a,b] where a denotes the

cost for associated pixel to take the label 0, and b denotes the cost for associated pixel to take the label 1.

The goal is to detect the foreground from the background using maxflow/mincut algorithm. Let the foreground be denoted by the label 0, and let the background be denoted by label 1. The unary costs for every pixel is given in the table given in Fig. 2. We use the standard Potts model for the pairwise costs. We use Potts pairwise model and it is given by $\{\theta_{x_i x_j}^{lm} = 0\ if\ l = m,\ and\ 1\ otherwise\}$ for adjacent pixels x_i and x_j. Write down the energy function and also show the graph that would be used for the 2-label segmentation. There is no need to solve the problem using maxflow/mincut algorithm, which any of-the-shelf solver can do.

[20 points]

6. Consider a multi-label problem with 2 variables y_1 and y_2 each taking 3 states $\{1,2,3\}$. Let the unary terms be given by $\{\theta_{y_1}^{l} = \{[l = 1] \rightarrow 0.5, [l = 2] \rightarrow 1.5, [l = 3] \rightarrow 1.0\},\ and\ \{\theta_{y_2}^{l} = \{[l = 1] \rightarrow 1.5, [l = 2] \rightarrow 1.5, [l = 3] \rightarrow 0.0\}$. We use Potts pairwise model and it is given by $\{\theta_{y_1 y_2}^{lm} = 0\ if\ l = m,\ and\ 1\ otherwise\}$.

   (a) Show the iterations in alpha-expansion till it converges. Note that you will construct a Boolean function in every iteration. You will manually compute the solution for every maxflow/mincut step.
   (b) Show the iterations in alpha-beta swap till it converges. Note that you will construct a Boolean function in every iteration. You will manually compute the solution for every maxflow/mincut step.

[40 points]