Abstract Interpretation

Static Analysis section, **Lecture 19**

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CS 6110, U of Utah

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Class Progress

Logical reasoning

Program logics

Static analysis

Online lecture for the rest of the term

Will try to continue as much as possible without change

Attendance cancelled: not taken, everyone marked present

Assignment 5 **cancelled**. Final presentation **virtual**.

Record final presentation; watch others' videos

Write and respond to written questions on presentation

Video Lecture

Download **Zoom app**, join this meeting at **normal time**:

232-421-488

Hold lectures, do exercises, Q&A as normal:

Stay muted; hold space bar to temporarily unmute

Raise hand to ask a question; I'll call on you } **Participants** panel

Technical support over chat with Manasij

Lectures **recorded**, re-watch / stream on Youtube

Project

Due dates **not changing**, project still on

Milestone II in class; final presentation recorded-only

- Prerecord as backup in case of connection issues
- Live Q&A in class for the Milestone II presentations
- Written Q&A for the final presentations (10% of grade)

Collaborate online for group projects

- Schedule a **1 hour weekly meeting** or more
- “Pair programming” for presentations works well

Schedule

Absences, extensions granted freely

- Still expecting achievement of most or all project goals
- Willing to do "**incomplete**"s for final projects, though

Some **reorganization of the schedule** expected

- Since final project presentations online, extra time

Your health is more important than class

- Please request absences, extensions in case of illness
- Child care, elderly care, and sick care all good reasons

A Note

No one knows what is going on or how to respond

- The administration is giving contradictory advice
- New guidance comes out daily

Currently, voting to make **grading pass/fail**

- May or may not pass; don’t get your hopes up

I’ll do what I can to make things right

- Please be flexible with me, but I’ll return the favor
- Don’t expect much coordination from the U as a whole

Another Note

My planning has been totally thrown off

- Let’s salvage what we can out of the semester
- Expect less prepared, less-polished lectures

Happily, **most essential content done**

- If you’ve learned how Dafny works and how to use it, I’m happy
- Learned **one** method of verification top-to-bottom

Refresher

Long spring break, huh?

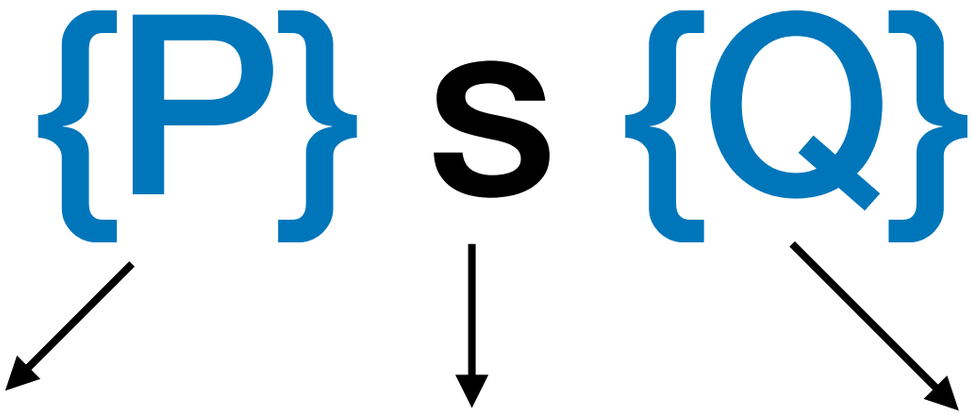
Verification

Consider verifying a **binary search**:

```
def bsearch(l, x):                          { sorted(l) }
   i, j = 0, len(l) - 1
   if j == -1 or x < l[i] or l[j] < x:
      return -1
   while j - i > 1:
      m = (i + j) / 2
      if l[m] < x: i = m
      elif l[m] > x: j = m
      else: return m
   if l[i] == x: return i
   elif l[j] == x: return j
   else: return -1
                                            { l[return] == x or return == -1 }
```

What We Learned

Add **pre-/post-conditions** to specify behavior



If **P** true of a state, and **s** executed, then **Q** true after

Each **{P} s {Q}** is a **logical statement**

Weakest preconditions systematically generate that statement

Verification

Consider verifying a **binary search**:

```
def bsearch(l, x):                          { sorted(l) }
   i, j = 0, len(l) - 1
   if j == -1 or x < l[i] or l[j] < x:
      return -1                             { 0 <= i <= j < len(l) }
while j - i > 1:                            { l[i] <= x <= l[j] }
   m = (i + j) / 2                          { DECREASES j - I }
   if l[m] < x: i = m
   elif l[m] > x: j = m
   else: return m                           { (l[i] == x and Q) or … }
if l[i] == x: return i
elif l[j] == x: return j
else: return -1                             { l[return] == x or return == -1 }
```

Statement Types

Loops as a form of **infinite statement**

- **Invariants** a short-hand for verifying that statement
- **Measures** for proving a loop terminates

Functions for **modular bits of code**

- **Reusing** function pre-/post-conditions at call sites
- **Measures** for proving recursive functions terminate

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else: return -1                             { l[return] == x or return == -1 }
```

Verification

Reasoning about the verification condition

```
[redacted] ->
([redacted] or [redacted] or [redacted])
 and ([redacted] or [redacted])) or
(not ([redacted] or [redacted] or [redacted])
 and ([redacted] and [redacted])
 and (([redacted] and ([redacted] and [redacted] -> …)
```

Boolean structure uses DPLL(T) algorithm

Put into **conjunctive form**: AND of ORs

Guess boolean value, simplify and infer new ones

Result is a **query** containing true theory terms

Verification

Reasoning about the verification condition

```
[redacted] ->
([redacted] or [redacted] or [redacted])
 and ([redacted] or [redacted])) or
(not ([redacted] or [redacted] or [redacted])
 and ([redacted] and [redacted])
 and (([redacted] and ([redacted] and [redacted] -> …)
```

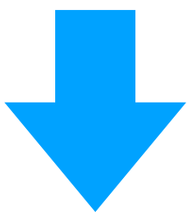
```
sorted(l) and ~X and ~Y and (X or ~A or ~B) and (~X or A) and …
```

```
sorted(l) and l[0] < x and 0 < len(l) and …
```

Verification

Reasoning about the query

```
sorted(l) and l[0] < x and 0 < len(l) and …
```



```
sorted(l) and l0 < x and l0 = l[0] and 0 < lenl and …
```

Integers

```
l0 < x and 0 < lenl and …
```

Arrays

```
sorted(l) and l0 = l[0] and …
```

Split the query into domain-specific queries

Verification

Reasoning about the domain-specific query

```
l0 < x and 0 < lenl and l0 >= x and …
```

Variable elimination for integer reasoning

Pick a variable x to eliminate

Group equations by x on the left or right

Form all pairs of equations, eliminating x

```
l0 <= l0 - 1
```

Staying Safe

Don’t get Coronavirus

Avoiding Illness

Keep a safe distance (6 feet) from other people

- Stay away from crowds; work from home if possible
- Shop, buy groceries less often (buy in bulk)
- Avoid large social outings (including for your children)

Keep clean; wash hands, wear gloves

- Virus spreads by **contact** (hand-shakes) or **coughing**
- Wash hands for **20 seconds** using soap
- Wear a mask or **cover your coughs** if you are sick

Symptoms

Mild cases (80%) have the usual flu symptoms

- Fever / high temperature
- Coughing, usually a dry cough
- Shortness of breath and general tiredness

Severe cases (20%): emergency medical attention (911)

- Difficulty breathing; bluish lips or face
- Pain or pressure in the chest
- Confusion and listlessness

Medical Care

Call your doctor if you think you have it

- You've recently travelled to Europe, China, Korea, or Iran
- You've been in contact with someone sick
- **Call before getting care** to protect others

Mild cases usually pass without issue

- Drink lots of liquids, don't skip meals, get rest
- Track and record your temperature regularly
- Isolate yourself for 3 days after the fever passes

More Information

coronavirus.gov

Scaling Up

What makes the process slow?

Abstract Interpretation

New goal of **scalability** for analyses

Approach: simple but specific over slow but general

Simple universe of predicates on program states

Loops analyzed by repeatedly re-analyzing

Lattice structure characterizes each analysis

Data structure for predicates, “or” method, finite height

Scalability

Process described so far can be **very slow**:

- Weakest preconditions **duplicate code**: exponential in code
- SAT solving is **exponential in formula**
- Result is **doubly-exponential**!

Process described so far can be **very general**:

- Limited only by **solver capabilities**
- Can describe **most correctness** and performance specs
- Writing specs is its own **time-consuming** task!

Alternative

Replace **slow, general** specs with **fast, specialized** ones

Array bounds

Nullness

Format validation

Invalid states

String escaping

Non-negativity

Replace **slow, general** specs with **fast, specialized** ones

Array bounds

Index non-negative

Index less than length

Non-negativity follows **simple rules**

Simplified conditions

Conditions: which variables **known to be non-negative**

Need to know: **which expressions** produce non-negative results

$$(\geq 0) + (\geq 0) \rightsquigarrow (\geq 0) \qquad (\geq 0) \times (\geq 0) \rightsquigarrow (\geq 0) \qquad (\geq 0) \div (\geq 0) \rightsquigarrow (\geq 0)$$

```
{ i ≥ 0 && j ≥ 0 }
m = (i + j) / 2
{ i ≥ 0 && j ≥ 0 && m ≥ 0 }
l[m]  ⟵ valid
```

Simple rules instead of solver queries: fast

Propagating Conditions

Simplified conditions **too general**; over-approximate

Make sure **rules for propagating** are also over-approximate

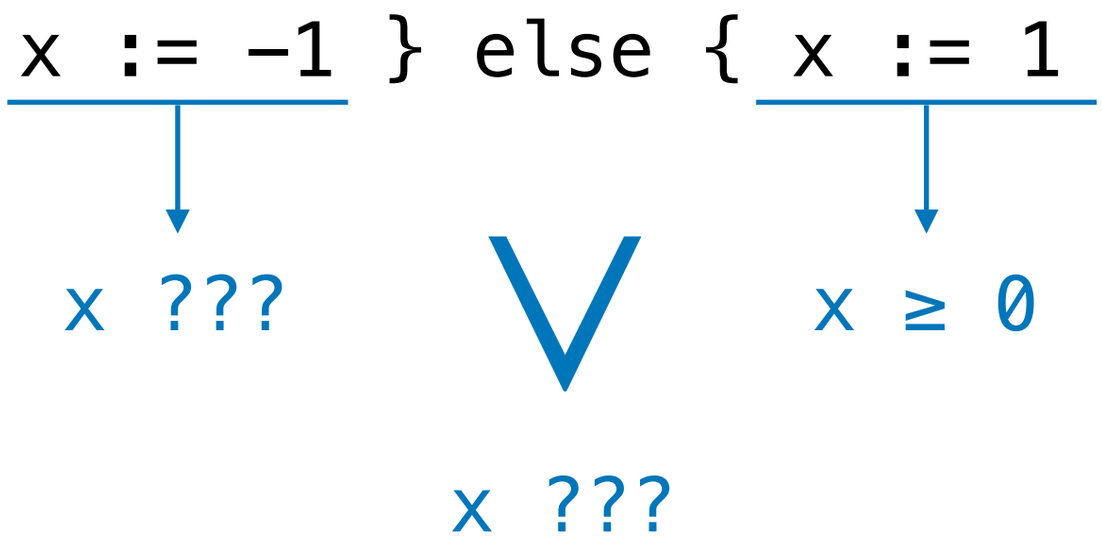
$$\{ P \}\ \texttt{skip}\ \{ P \}$$

$$\{ P \}\ s\ \{ Q \} \land \{ Q \}\ t\ \{ R \} \rightarrow \{ P \}\ s; t\ \{ R \}$$

$$e \geq 0 \rightarrow \{ P(x) \}\ x := e\ \{ x \geq 0 \}$$

Propagating Conditions

If statements require **“or”** for simplified conditions

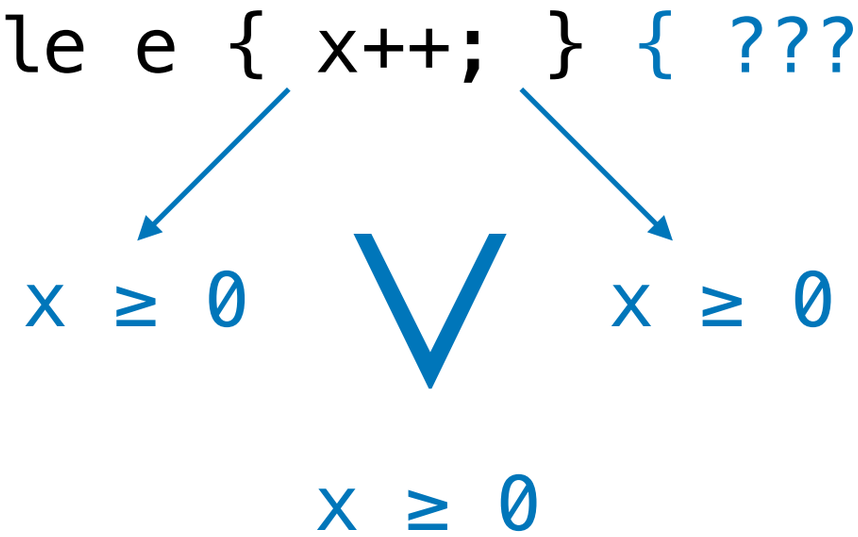


The conditional could also be over-approximated

Called “**flow-sensitive**” analysis; ignored here

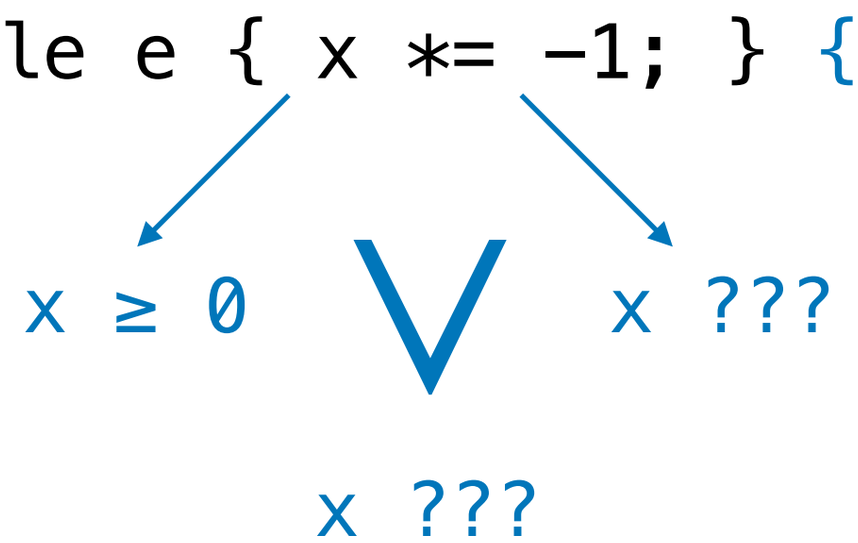
Loops

If statements require **“or”** for simplified conditions



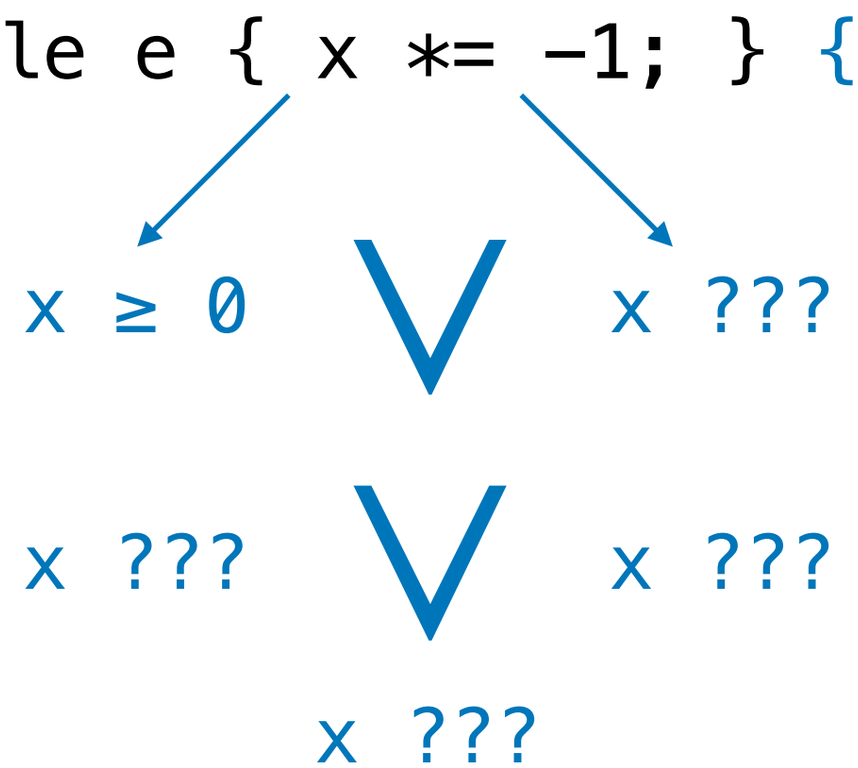
Loops

If statements require **“or”** for simplified conditions



Loops

If statements require **“or”** for simplified conditions



Repeat loop analysis until result stops changing

Why it works

Each step **over-approximates** program state

- If warning skipped, definitely no issue

However, **some specifications** cannot be stated

- Specifically restricted set of conditions

Analysis of loops **repeated** until fix-point

- Lattice must have **finite height** for analysis to terminate

Abstract Interpretation

A general approach to simplified conditionals

Generalizing

Create a **simple universe** of simplified conditions

Conditions for 1) **values**, and 2) **program states**

Non-negative

Unknown

Over-approximating **“or” operation** on conditions

Also need a least-specific “unknown” state

Example

Track **sign** of values & variables

Positive

Zero

Negative

Non-positive

Non-negative

Non-zero

Unknown

OR	+	0	-	!+	!-	!0	?
+							
0							
-							
!+							
!-							
!0							
?							

Example

Track **sign** of values & variables

Operation rules:

```
(+) + (+) = (+)
(+) + (0) = (+)
(+) + (−) = (?)
```

…

```
(+) * (+) = (+)
(−) * (−) = (+)
```

…

Constant rules:

```
0 is (0)
1, 2, … is (+)
−1, −2, … is (−)
```

Conditional rules:

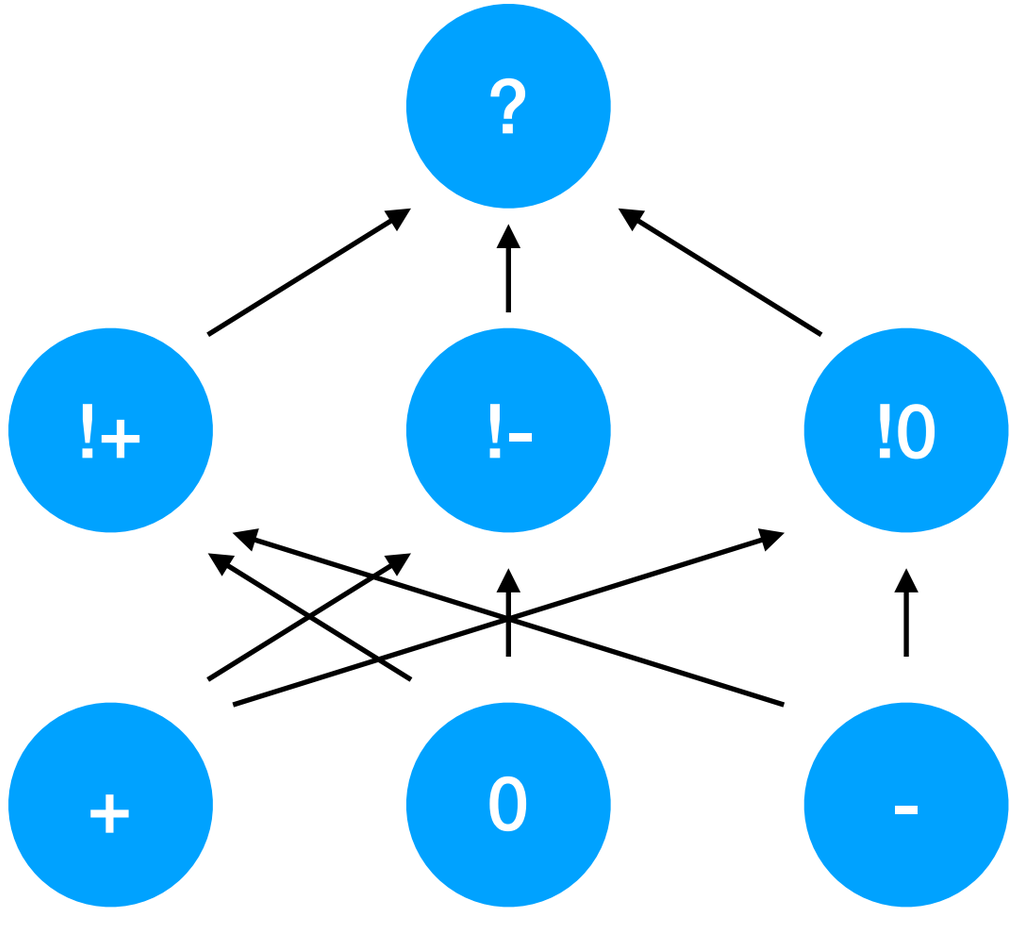
```
(+) < x → x is (+)
(−) < (+) = True
```

…

Example

OR operation produces a **graph structure**

A → B when (A OR B) = B; structure called a **lattice**



Exercise

Define an analysis for **evenness/oddness** of integers

Analysis

Represent predicates by a **data structure**

- The “or” function is a **method** on the structure

Define **rules for propagating** predicates over statements

- Also rules for assigning predicates to expressions
- Analysis of loops requires repeating until fix-point

Finally, **issue warnings** on certain operations

- Usually, check for certain predicates on function arguments

Next class:

Common Domains

To do:

- ☐ Course feedback
- ☐ Work on project

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RELATIONS

MULTIPLE

VARIABLES

INTERVALS

WIDENING

NARROWING

EXAMPLE

ARRAY

BOUNDS

Next class:

Common Domains

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