Lecture Notes: Risk Analysis and Expectation

Suppose a student named *Shylock*, "*Shy*" for short, had won a lot of money in a heavy poker game, say $10,000 over the course of a long weekend. His plan is to make subprime loans to other students who would not qualify for such a loan at an established load institution. For purposes of illustration, we further assume that the loans are to be repaid in a lump sum, a balloon payment, in exactly one year.

First *Shy* must accurately assess the borrower's likelihood of repaying the loans. Let R_i be the *risk* that student S_i will default on a loan, so the likelihood of repayment is then $1.0 - R_i$. If there is a 25% risk that a student will default, then the likelihood of repayment is 75% .

Shy knows that diversification is important for spreading the risk across several transactions, so, at this point he has made 3 loans to $S_1, S_2, and\ S_3$, with *risk* factors of $R_1, R_2, and\ R_3$ of 30%, 40% and 20% , respectively, in the *principal* amounts $P_1, P_2, and\ P_3$ of $1000, $3,000 and $2,500, respectively.

Now, calculate how *Shy* should structure these loan transactions so that he statistically covers the risk involved and then adds in a *profit* factor of 15% . First we take into account the risk by charging a statistical *risk factor* to compensate for the subprime quality of the loan.

1. Without any other considerations, what is the expected value of L_1, the loan issued to student S_1 ? This is straightforwardly seen as

$$\begin{aligned} E(L_1) &= (1 - R_1) \times P_1 \\ &= (1 - 0.30) \times \$1{,}000 \\ &= \$700 \end{aligned}$$

   With analogous calculations, it follows that,

$$\begin{aligned} E(L_2) &= (1 - R_2) \times P_2 \\ &= (1 - 0.40) \times \$3{,}000 \\ &= \$1{,}800 \end{aligned}$$

   and,

$$\begin{aligned} E(L_3) &= (1 - R_3) \times P_3 \\ &= (1 - 0.20) \times \$2{,}500 \\ &= \$2{,}000 \end{aligned}$$

2. Now *Shy* decides that he wants to buy a car, so he goes to a dealer to look at used cars, where he expects to finance the car using his "loan portfolio" as collateral. How should be value of his portfolio of 3 loans be valuated? Although he has lent

out a total of $6,500 in principal, without further adjustments his portfolio of outstanding loans is worth only its expectation,

$$\sum_{i=1}^{3}(1 - R_i) \times P_i = \$4{,}500$$

Therefore, the car dealer will only consider his portfolio worth $4,500, not $6,000 of original principal in collateral value. This illustrates that he losing net worth unless he adjusts for the risk.

3. From the above we see that a *risk terms* of $RT_1 = \$300$, $RT_2 = \$1{,}200$, and $RT_3 = \$500$, are indicated, for $L_1, L_2\ and\ L_3$, respectively, simply to compensate for the assumed risk that the loans might not be repaid. This is enough to “break even” statistically, so that we can assume that the expected value for L_1 is now $700 + $300 = $1,000, the original principal; for L_2 , $1,800 + $1,200 = $3,000, the original principal; and for L_3 , $2000 + $500 = $2,500, the original principal.

4. *Shy* still has to include a 15% profit term in his loan structure, a figure which he assumes will also cover the mild affects of inflation today. Hence, additional terms *service terms* of $S_1 = \$150$, $S_2 = \$450$ and $S_3 = \$375$, need to be inserted in Loans $L_1, L_2\ and\ L_3$, respectively. In view of the profit term, the loans will look like, $L_i = P_i + RT_i + S_i$.

5. To summarize all of the above,

   i. $L_1 = \$1{,}000 + \$300 + \$150 = \$1{,}450$

   ii. $L_2 = \$3{,}000 + \$1{,}200 + \$450 = \$4{,}650$

   iii. $L_3 = \$2{,}500 + \$500 + \$375 = \$3{,}375$

□