

Lecture Note #10: Binomial recursive relationship

We derive the following recursive relationship for binomial coefficients,

$$k \binom{n}{k} = n \binom{n-1}{k-1} \quad (1.1)$$

Derivation:

$$k \binom{n}{k} = k \left(\frac{1 \cdot 2 \cdot \dots \cdot n}{(1 \cdot 2 \cdot \dots \cdot k)(1 \cdot 2 \cdot \dots \cdot (n-k))} \right)$$

Factoring out an n from the numerator and canceling a k in the denominator, we get

$$= n \left(\frac{1 \cdot 2 \cdot \dots \cdot (n-1)}{(1 \cdot 2 \cdot \dots \cdot (k-1))(1 \cdot 2 \cdot \dots \cdot (n-k))} \right)$$

This now directly yields the sought after relationship, because $(n-1) - (k-1) = n-k$, as presented in the denominator. Therefore, we have derived the RHS, and shown,

$$k \binom{n}{k} = n \binom{n-1}{k-1}$$

□