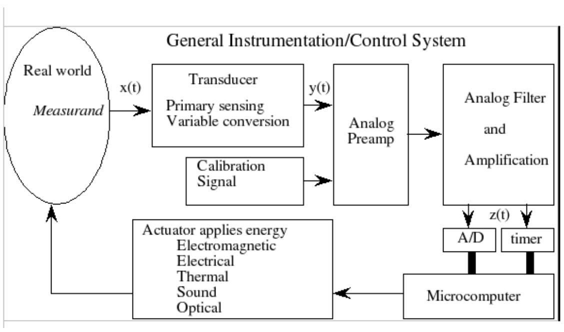
Today

- Data acquisition
- Digital filters and signal processing
  - Filter examples and properties
  - FIR filters
  - Filter design
  - Implementation issues
  - DACs
  - PWM

Data Acquisition Systems

- Many embedded systems measure quantities from the environment and turn them into bits
  - These are data acquisition systems (DAS)
  - This is fundamental
- Sometimes data acquisition is the main idea
  - Digital thermometer
  - Digital camera
  - Volt meter
  - Radar gun
- Other times DAS is mixed with other functionality
  - Digital signal processing
  - Networking, storage
  - Feedback control

Big Picture



Why Care About DAS?

- July 1983: Air Canada 143, a Boeing 767, runs out of fuel in mid-air, lands on "abandoned" runway
- Poorly soldered fuel level sensor + mistakes that defeated backup systems

Accuracy

- *Instrument accuracy* is the absolute error of the entire system, including transducer, electronics, and software
- Let x_{mi} be measured value and x_{ti} be the true value
- Average accuracy: $\frac{1}{n}\sum_{i=1}^{n}|x_{ti}-x_{mi}|$
- Average accuracy of reading: $\frac{100}{n}\sum_{i=1}^{n}\frac{|x_{ti}-x_{mi}|}{x_{ti}}$
- Average accuracy of full scale: $\frac{100}{n}\sum_{i=1}^{n}\frac{|x_{ti}-x_{mi}|}{x_{tmax}}$

More Accuracy

- Maximum error: $\max|x_{ti}-x_{mi}|$
- Maximum error of reading: $100\max\frac{|x_{ti}-x_{mi}|}{x_{ti}}$
- Maximum error of full scale: $100\max\frac{|x_{ti}-x_{mi}|}{x_{tmax}}$

Resolution

- Instrument resolution is the smallest input signal difference that can be detected by the entire system
  - May be limited by noise in either transducer or electronics
- Spatial resolution of the transducer is the smallest distance between two independent measurements
  - Determined by size and mechanical properties of the transducer

Precision

- Precision is number of distinguishable alternatives, n_x, from which result is selected
- Can be expressed in bits or decimal digits
  - 1000 alternatives: 10 bits, 3 decimal digits
  - 2000 alternatives: 11 bits, 3.5 decimal digits
  - 4000 alternatives: 12 bits, 3.75 decimal digits
  - 10000 alternatives: >13 bits, 4 decimal digits
- Range is resolution times precision: $r_x = \Delta x \ n_x$

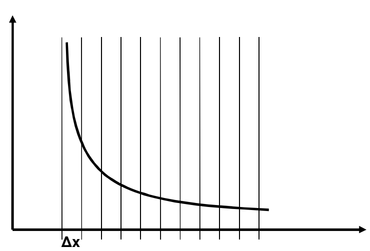
Reproducibility

- *Reproducibility* specifies whether the instrument has equal outputs given identical inputs over some time period
- Specified as full range or standard deviation of output results given a fixed input
- Reproducibility errors often come from transducer drift

ADC: How many bits?

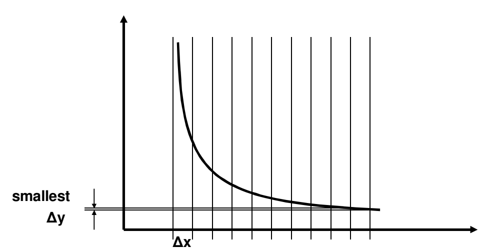
- Linear transducer case:
  - ADC resolution must be ≥ problem resolution
- Nonlinear transducer case:
  - Let x be the real-world signal with range r_x
  - Let y be the transducer output with range r_y
  - Let the required precision of x be n_x
  - Resolutions of x and y are Δx and Δy
  - Transducer response described by $y=f(x)$
  - Required ADC precision n_y (number of alternatives) is:
    - $\Delta x = r_x/n_x$
    - $\Delta y = \min \{ f(x + \Delta x) - f(x) \}$ for all x in r_x
  - Bits is ceiling($\log_2 n_y$)

ADC: How many bits?



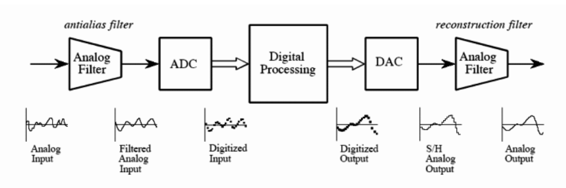
- ADC must be able to measure a change in voltage of the smallest Δy

ADC: How many bits?



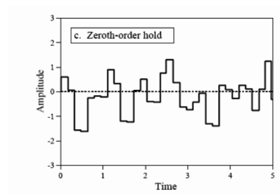
- ADC must be able to measure a change in voltage of the smallest Δy

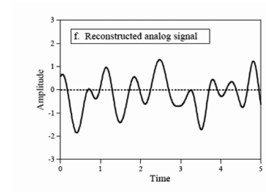
DSP Big Picture



Signal Reconstruction

- Analog filter gets rid of unwanted high-frequency components in the output





Data Acquisition

- Signal: Time-varying measurable quantity whose variation normally conveys information
  - Quantity often a voltage obtained from some transducer
  - E.g. a microphone
- Analog signals have infinitely variable values at all times
- Digital signals are discrete in time and in value
  - Often obtained by sampling analog signals
  - Sampling produces sequence of numbers
    - E.g. { ... , x[-2], x[-1], x[0], x[1], x[2], ... }
  - These are time domain signals

Sampling

- Transducers
  - Transducer turns a physical quantity into a voltage
  - ADC turns voltage into an *n*-bit integer
  - Sampling is typically performed periodically
  - Sampling permits us to reconstruct signals from the world
    - E.g. sounds, seismic vibrations
- Key issue: ***aliasing***
  - ***Nyquist rate***: 0.5 * sampling rate
  - Frequencies higher than the Nyquist rate get mapped to frequencies below the Nyquist rate
  - Aliasing cannot be undone by subsequent digital processing

Sampling Theorem

- Discovered by Claude Shannon in 1949:

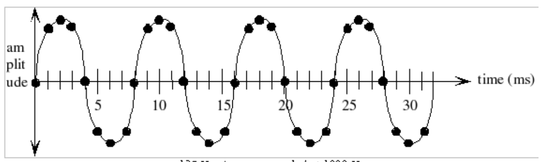
  A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the sampling frequency

- This is a pretty amazing result
  - But note that it applies only to discrete time, not discrete values

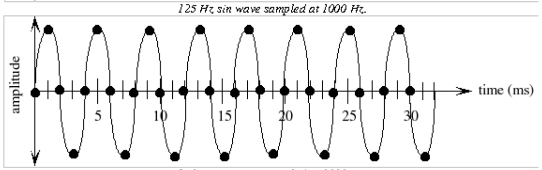
Aliasing Details

- Let N be the sampling rate and F be a frequency found in the signal
  - Frequencies between 0 and 0.5*N are sampled properly
  - Frequencies >0.5*N are aliased
    - Frequencies between 0.5*N and N are mapped to (0.5*N)-F and have phase shifted 180°
    - Frequencies between N and 1.5*N are mapped to f-N with no phase shift
    - Pattern repeats indefinitely
- Aliasing may or may not occur when N == F*2*X where X is a positive integer

No Aliasing

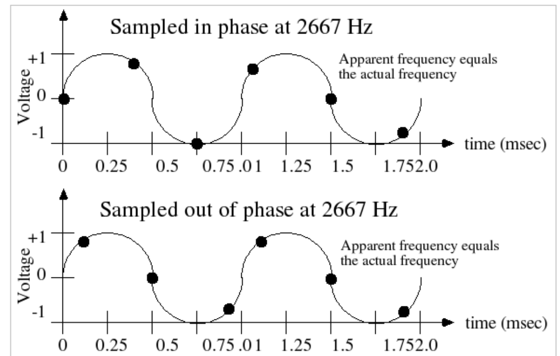


125 Hz sin wave sampled at 1000 Hz.

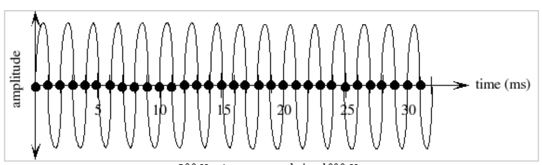


250 Hz sin wave sampled at 1000 Hz.

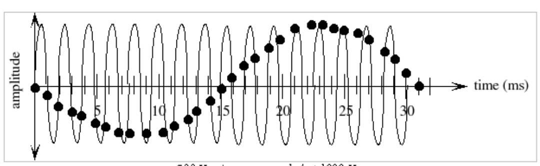
1 kHz Signal, No Aliasing



Aliasing



500 Hz sin wave sampled at 1000 Hz.

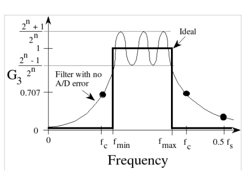


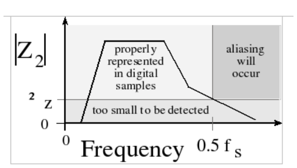
533 Hz sin wave sampled at 1000 Hz.

Avoiding Aliasing

1. **Increase sampling rate**
   - **Not a general-purpose solution**
     - **White noise is not band-limited**
     - **Faster sampling requires:**
       - **Faster ADC**
       - **Faster CPU**
       - **More power**
       - **More RAM for buffering**
2. **Filter out undesirable frequencies before sampling using analog filter(s)**
   - **This is what is done in practice**
   - **Analog filters are imperfect and require tradeoffs**

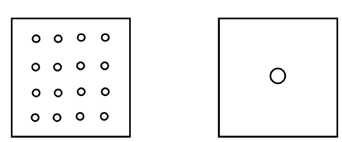
Signal Processing Pragmatics





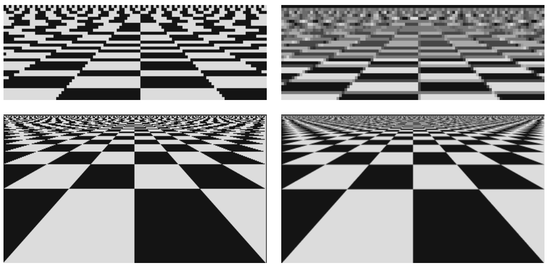
Aliasing in Space

- **Spatial sampling incurs aliasing problems also**
- **Example: CCD in digital camera samples an image in a grid pattern**
  - **Real world is not band-limited**
  - **Can mitigate aliasing by increasing sampling rate**



Samples **Pixel**

Point vs. Supersampling



Point sampling **4x4 Supersampling**

Digital Signal Processing

- **Basic idea**
  - Digital signals can be manipulated losslessly
  - SW control gives great flexibility
- **DSP examples**
  - Amplification or attenuation
  - Filtering – leaving out some unwanted part of the signal
  - Rectification – making waveform purely positive
  - Modulation – multiplying signal by another signal
    - E.g. a high-frequency sine wave

Assumptions

1. **Signal sampled at fixed and known rate f_s**
   - I.e., ADC driven by timer interrupts
2. **Aliasing has not occurred**
   - I.e., signal has no significant frequency components greater than $0.5*f_s$
   - These have to be removed before ADC using an analog filter
   - Non-significant signals have amplitude smaller than the ADC resolution

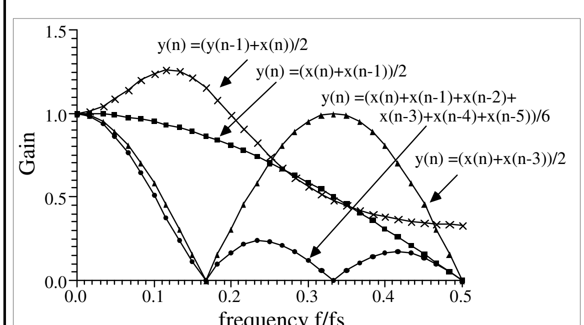
Filter Terms for CS People

- **Low pass – lets low frequency signals through, suppresses high frequency**
- **High pass – lets high frequency signals through, suppresses low frequency**
- **Passband – range of frequencies passed by a filter**
- **Stopband – range of frequencies blocked**
- **Transition band – in between these**

Simple Digital Filters

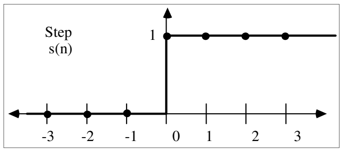
- **y(n) = 0.5 * (x(n) + x(n-1))**
  - Why not use x(n+1)?
- **y(n) = (1.0/6) * (x(n) + x(n-1) + x(n-2) + … + x(n-5))**
- **y(n) = 0.5 * (x(n) + x(n-3))**
- **y(n) = 0.5 * (y(n-1) + x(n))**
  - What makes this one different?
- **y(n) = median [x(n) + x(n-1) + x(n-2)]**

Gain vs. Frequency

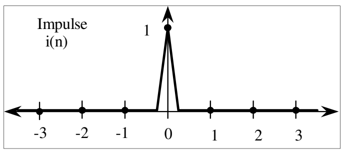


Useful Signals

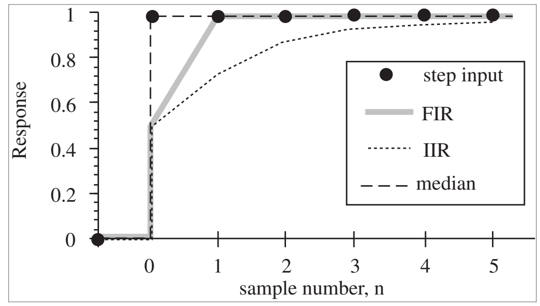
- **Step:**
  - **…, 0, 0, 0, 1, 1, 1, …**



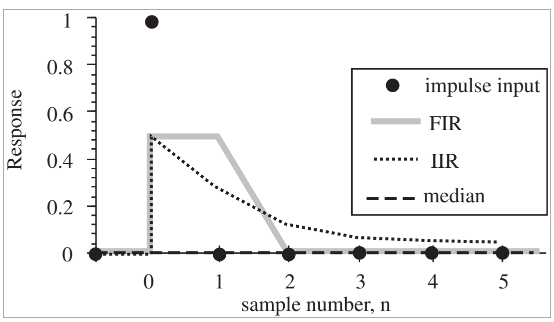
- **Impulse:**
  - **…, 0, 0, 0, 1, 0, 0, …**



Step Response



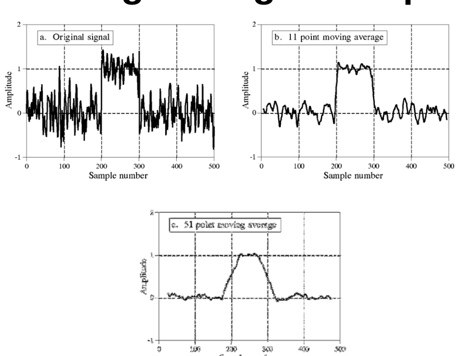
Impulse Response



FIR Filters

- **Finite impulse response**
  - **Filter "remembers" the arrival of an impulse for a finite time**
- **Designing the coefficients can be hard**
- **Moving average filter is a simple example of FIR**

Moving Average Example



FIR in C

```
SAMPLE fir_basic (SAMPLE input, int ntaps,
                  const SAMPLE coeff[],
                  SAMPLE z[])
{
  z[0] = input;
  SAMPLE accum = 0;
  for (int ii = 0; ii < ntaps; ii++) {
    accum += coeff[ii] * z[ii];
  }
  for (ii = ntaps - 2; ii >= 0; ii--) {
    z[ii + 1] = z[ii];
  }
  return accum;
}
```

Implementation Issues

- Usually done with fixed-point
- How to deal with overflow?
- A few optimizations
  - Put coefficients in registers
  - Put sample buffer in registers
  - Block filter
    - Put both samples and coefficients in registers
    - Unroll loops
  - Hardware-supported circular buffers
- Creating very fast FIR implementations is important

Filter Design

- Where do coefficients come from for the moving average filter?
- In general:
  1. Design filter by hand
  2. Use a filter design tool
- Few filters designed by hand in practice
- Filters design requires tradeoffs between
  1. Filter order
  2. Transition width
  3. Peak ripple amplitude
- Tradeoffs are inherent

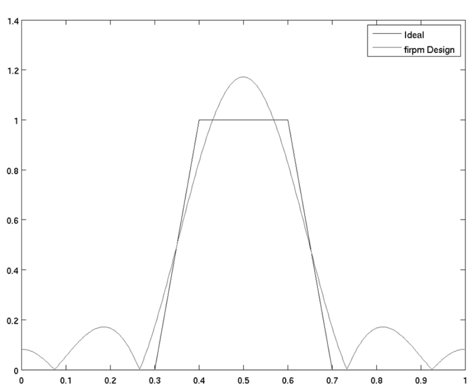
Filter Design in Matlab

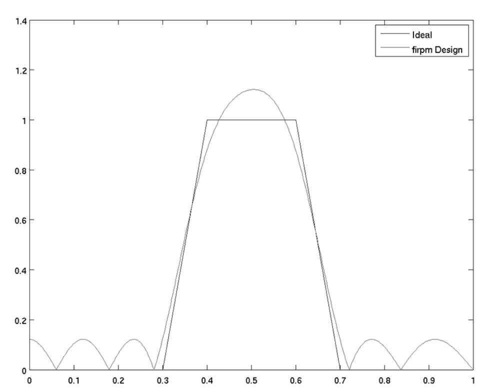
- Matlab has excellent filter design support
  - `C = firpm (N, F, A)`
  - N = length of filter - 1
  - F = vector of frequency bands normalized to Nyquist
  - A = vector of desired amplitudes
- `firpm` uses minimax – it minimizes the maximum deviation from the desired amplitude

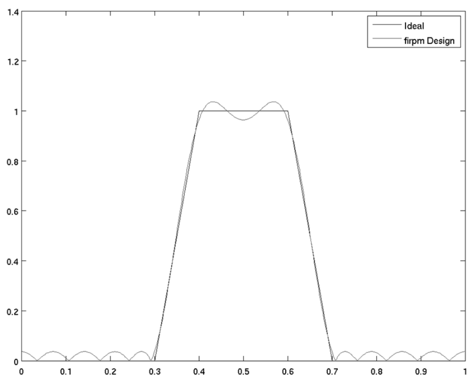
Filter Design Examples

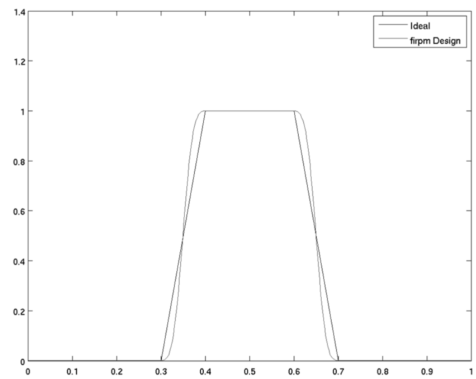
```
f = [ 0.0 0.3 0.4 0.6 0.7 1.0];
a = [   0   0   1   1   0   0];
fil1 = firpm( 10, f, a);
fil2 = firpm( 17, f, a);
fil3 = firpm( 30, f, a);
fil4 = firpm(100, f, a);
```

```
fil2 =
  Columns 1 through 8
-0.0278 -0.0395 -0.0019 -0.0595  0.0928  0.1250 -0.1667 -0.1985
  Columns 9 through 16
 0.2154  0.2154 -0.1985 -0.1667  0.1250  0.0928 -0.0595 -0.001
  Columns 17 through 18
-0.0395 -0.0278
```









Testing an FIR Filter

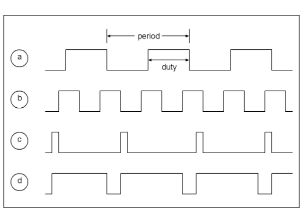
- **Impulse test**
  - **Feed the filter an impulse**
  - **Output should be the coefficients**
- **Step test**
  - **Feed the filter a test**
  - **Output should stabilize to the sum of the coefficients**
- **Sine test**
  - **Feed the filter a sine wave**
  - **Output should have the expected amplitude**

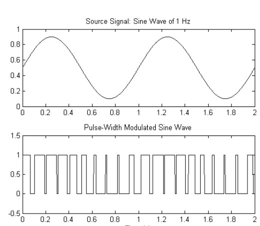
Digital to Analog Converters

- **Opposite of an ADC**
- **Available on-chip and as separate modules**
  - **Also not too hard to build one yourself**
- **DAC properties:**
  - **Precision: Number of distinguishable alternatives**
    - **E.g. 4092 for a 12-bit DAC**
  - **Range: Difference between minimum and maximum output (voltage or current)**
  - **Speed: Settling time, maximum output rate**
- **LPC2129 has no built-in DACs**

Pulse Width Modulation

- **PWM answers the question: How can we generate analog waveforms using a single-bit output?**
  - **Can be more efficient than DAC**





PWM

- **Approximating a DAC:**
  - **Set PWM period to be much lower than DAC period**
  - **Adjust duty cycle every DAC period**
- **Important application of PWM is in motor control**
  - **No explicit filter necessary – inertia makes the motor its own low-pass filter**
- **PWM is used in some audio equipment**

Summary

- Filters and other DSP account for a sizable percentage of embedded system activity
- Filters involve unavoidable tradeoffs between
  - Filter order
  - Transition width
  - Peak ripple amplitude
- In practice filter design tools are used
- We skipped all the theory!
  - Lots of ECE classes on this