Lecture Set 10

Visible Surface Determination

CS5600 *Computer Graphics*

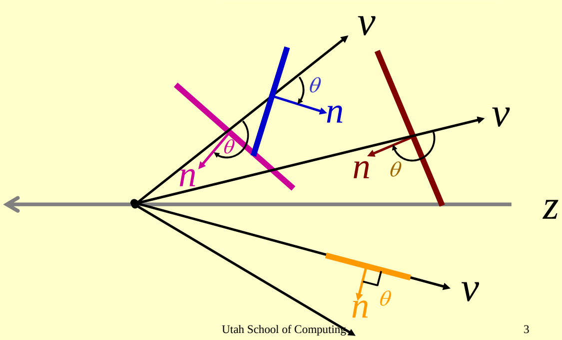
From Rich Riesenfeld

Spring 2013

Class of Algorithms

- Object (Model) Space Algorithms
  - Work in the model data space
- Image Space Algorithms
  - Work in the projected space
  - Most common VSD domain

Back Face Culling: Object Space



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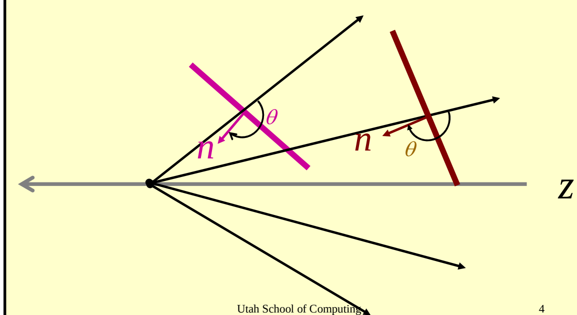
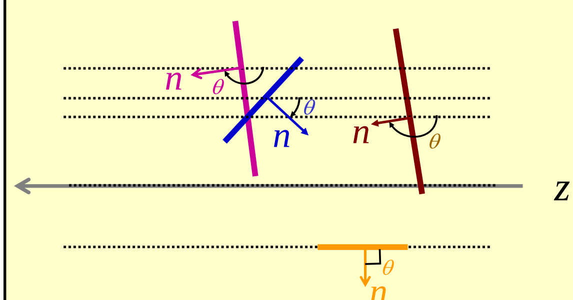


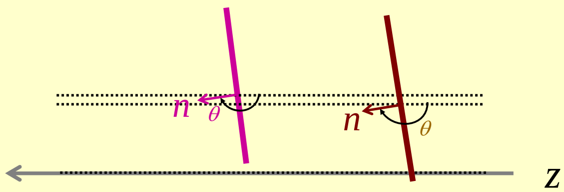
Back Face Culling Test

- For Object Space look at sign of $v \cdot n$
- For Image Space look at sign of n_z

Back Face Culling: Image Space

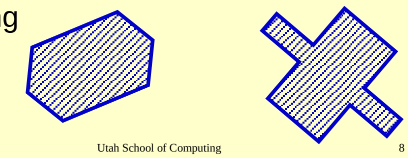


Back Face Culling: Image Space

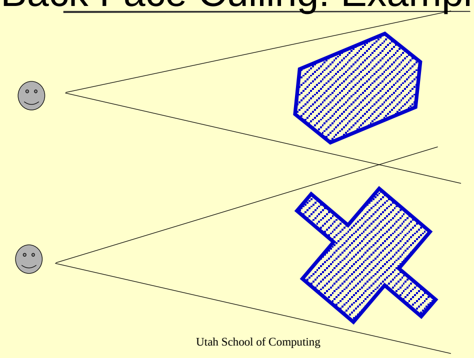


Back Face Culling

- Completes the job for convex polyhedral objects
- Nonconvex objects need additional processing beyond back face culling



Back Face Culling: Examples



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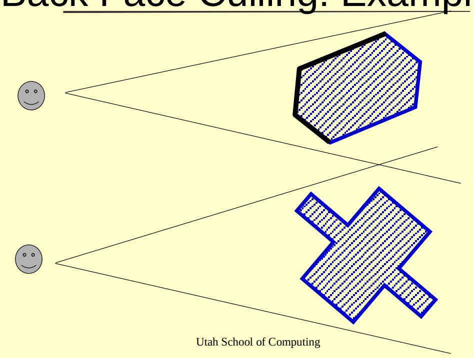
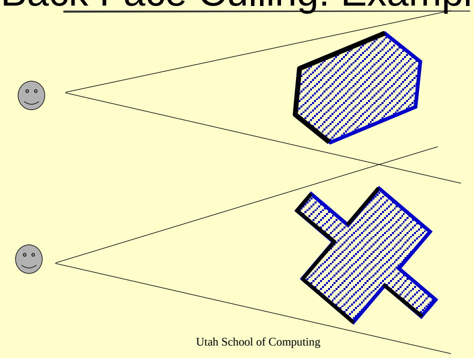
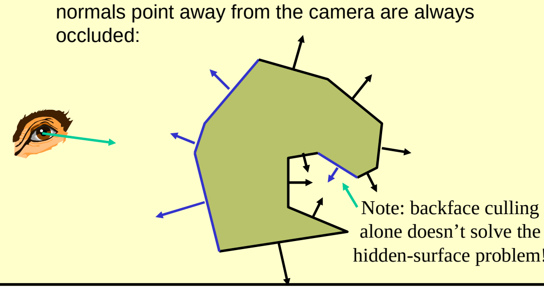


Back Face Culling: Examples



Back-Face Culling

- On the surface of a closed manifold, polygons whose normals point away from the camera are always occluded:



Back-Face Culling

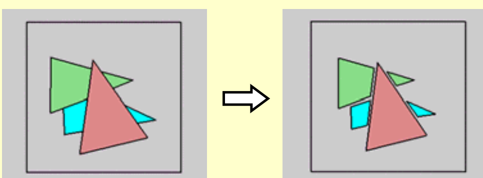
- Not rendering backfacing polygons improves performance
  - *By how much?*
    - *Reduces by about half the number of polygons to be considered for each pixel*

Silhouettes

- For Object Space $v \cdot n = 0$
- For Image Space $n_z = 0$

Occlusion

- For most interesting scenes, some polygons will overlap:



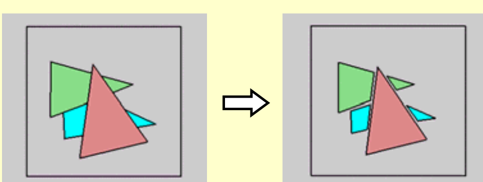
- To render the correct image, we need to determine which polygons *occlude* which

Painter's Algortihm

- How do painter's solve this?

Painter's Algorithm

- Simple approach: render the polygons from back to front, "painting over" previous polygons:



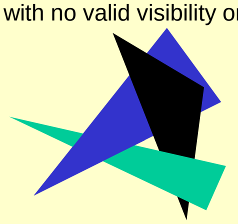
  - Draw blue, then green, then orange
- *Will this work in the general case?*

Painter's Algortihm

- How do painter's solve this?
- Sort the polygons in depth order
- Draw the polygons back-to-front
- QED

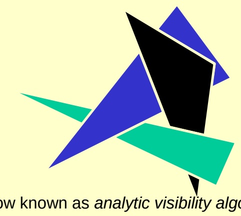
Painter's Algorithm: Problems

- *Intersecting polygons* present a problem
- Even non-intersecting polygons can form a cycle with no valid visibility order:



Analytic Visibility Algorithms

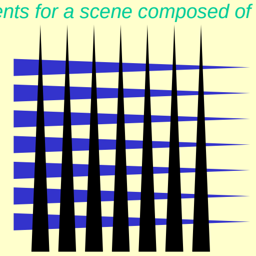
- Early visibility algorithms computed the set of visible polygon *fragments* directly, then rendered the fragments to a display:



  – Now known as *analytic visibility algorithms*

Analytic Visibility Algorithms

- *What is the minimum worst-case cost of computing the fragments for a scene composed of* n *polygons?*
- Answer: O(n^2)



Analytic Visibility Algorithms

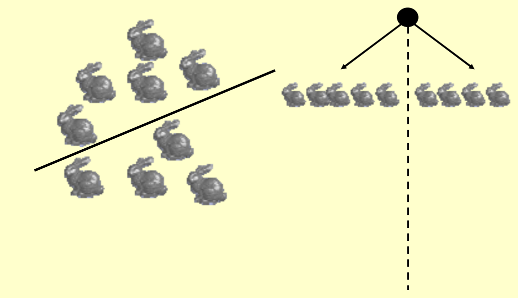
- So, for about a decade (late 60s to late 70s) there was intense interest in finding efficient algorithms for *hidden surface removal*
- We'll talk about two:
  – *Binary Space-Partition (BSP) Trees*
  – *Warnock's Algorithm*

Binary Space Partition Trees (1979)

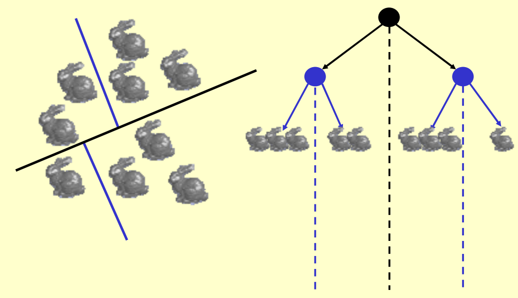
- BSP tree: organize all of space (hence *partition*) into a binary tree
  – *Preprocess*: overlay a binary tree on objects in the scene
  – *Runtime*: correctly traversing this tree enumerates objects from back to front
  – Idea: divide space recursively into half-spaces by choosing *splitting planes*
    - Splitting planes can be arbitrarily oriented
    - Notice: nodes are always convex

BSP Trees: Objects

BSP Trees: Objects



BSP Trees: Objects



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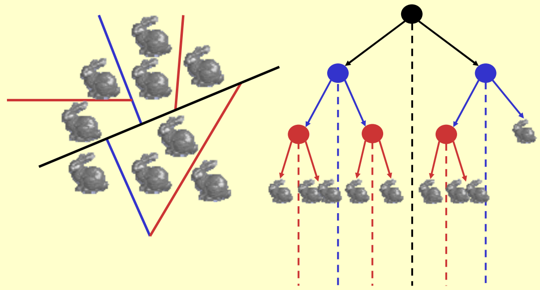

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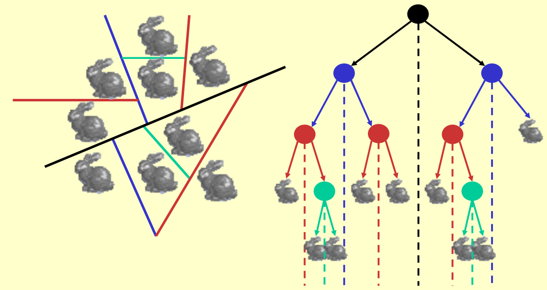
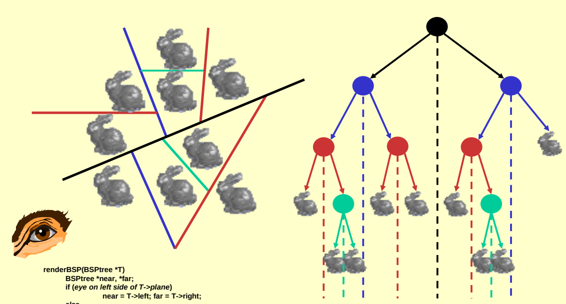


Rendering BSP Trees

```
renderBSP(BSPtree *T)
  BSPtree *near, *far;
  if (T is a leaf node)
     renderObject(T)
  else {
     if (eye on left side of T->plane)
         near = T->left; far = T->right;
     else
         near = T->right; far = T->left;
     renderBSP(far);
     renderBSP(near);
  }
```

Rendering BSP Trees



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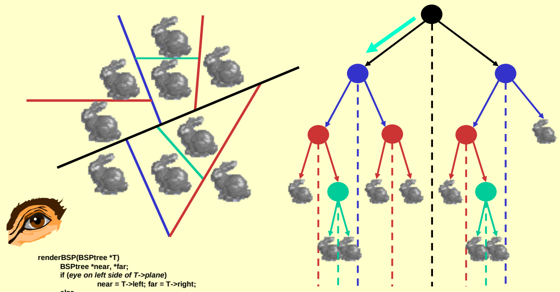

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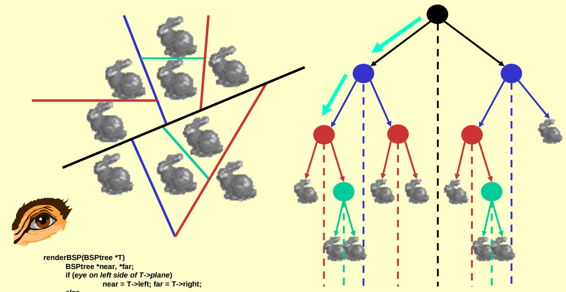

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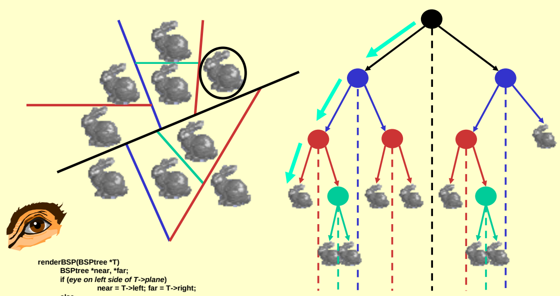

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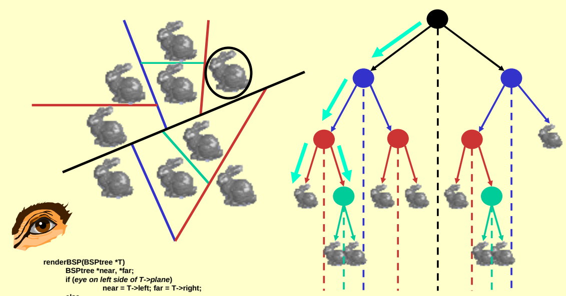

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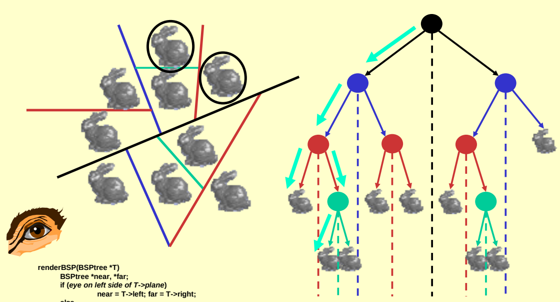

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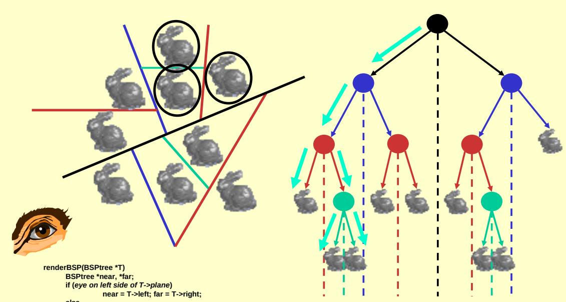

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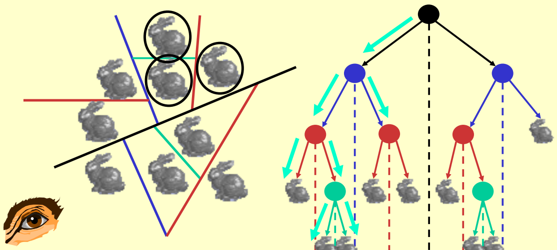

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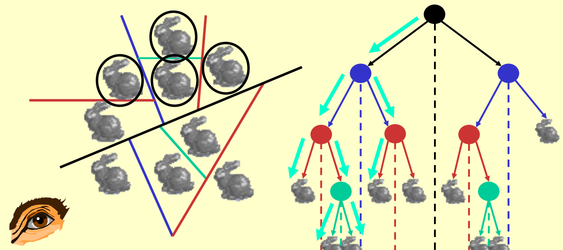

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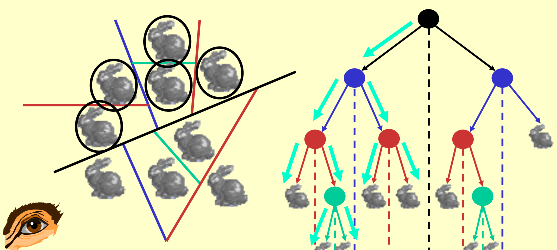

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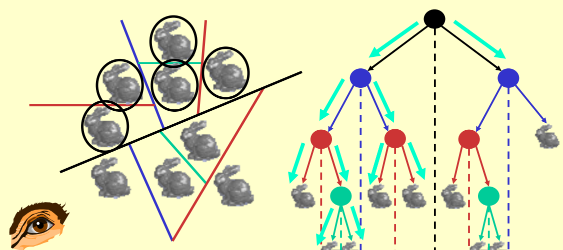

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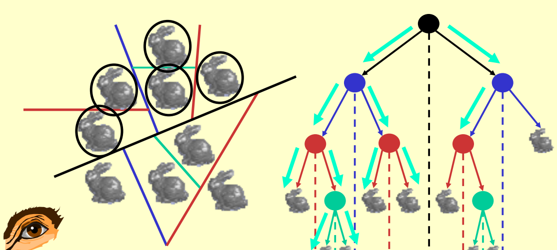

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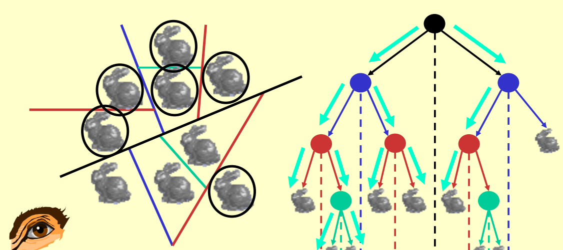

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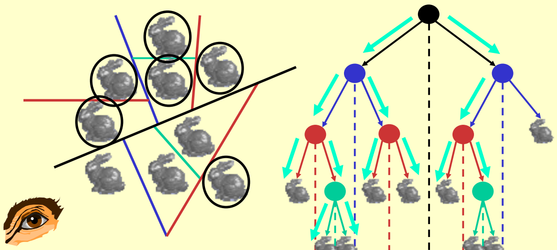

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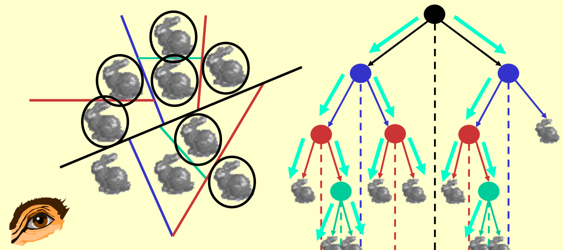

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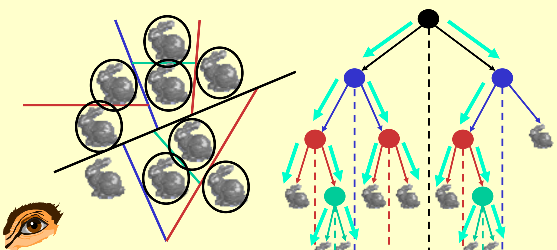

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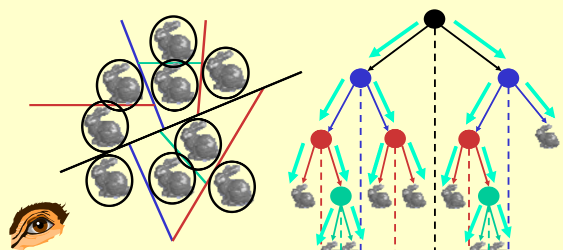

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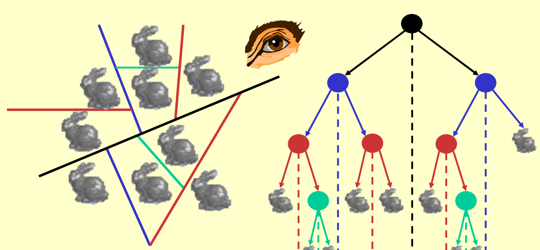

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3D Polygons: BSP Tree Construction

- Split along the plane containing any polygon
- Classify all polygons into positive or negative half-space of the plane
  - If a polygon intersects plane, split it into two
- Recurse down the negative half-space
- Recurse down the positive half-space

Polygons: BSP Tree Traversal

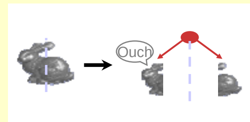
- Query: given a viewpoint, produce an ordered list of (possibly split) polygons from back to front:

```
BSPnode::Draw(Vec3 viewpt)
   Classify viewpt: in + or - half-space of node->plane?
   /* Call that the "near" half-space */
        farchild->draw(viewpt);
        render node->polygon; /* always on node->plane */
        nearchild->draw(viewpt);
```

- Intuitively: at each partition, draw the stuff on the farther side, then the polygon on the partition, then the stuff on the nearer side

Discussion: BSP Tree Cons

- No bunnies were harmed in my example
- But what if a splitting plane passes through an object?
  - Split the object; give half to each node:



  - Worst case: can create up to O(n^3) objects!

BSP Demo

- Nice demo:

http://www.symbolcraft.com/graphics/bsp/

Summary: BSP Trees

- Pros:
  - Simple, elegant scheme
  - Only writes to framebuffer (i.e., painters algorithm)
    - Once very popular for video games (but getting less so)
    - Widely used in ray-tacing
- Cons:
  - Computationally intense preprocess stage restricts algorithm to static scenes
  - Worst-case time to construct tree: O(n^3)
  - Splitting increases polygon count
    - Again, O(n^3) worst case

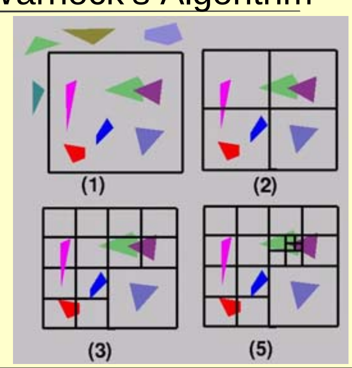
Warnock's Algorithm (1969)

PIXAR uses a similar scheme

- Elegant scheme based on a powerful general approach common in graphics: *if the situation is too complex, **subdivide***
  - Start with a *root viewport* and a list of all primitives
  - Then recursively:
    - Clip objects to viewport
    - If number of objects incident to viewport is zero or one, visibility is trivial
    - Otherwise, subdivide into smaller viewports, distribute primitives among them, and recurse

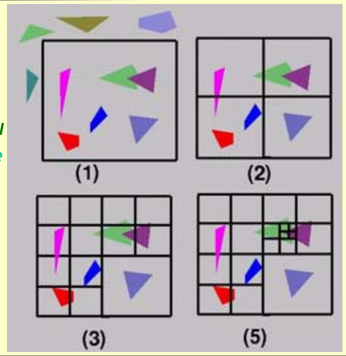
Warnock's Algorithm

- *What is the terminating condition?*
- *How to determine the correct visible surface in this case?*



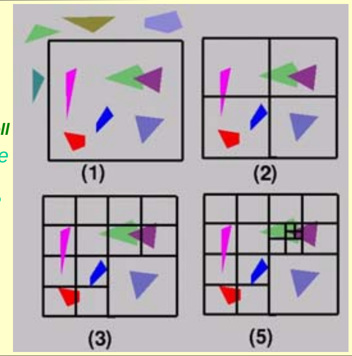
Warnock's Algorithm

- *What is the terminating condition?*
  - ***One polygon per cell***
- *How to determine the correct visible surface in this case?*



Warnock's Algorithm

- *What is the terminating condition?*
  - ***One polygon per cell***
- *How to determine the correct visible surface in this case?*
  - ***Cell = single pixel***



Warnock's Algorithm

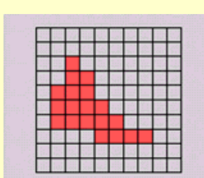
- Pros:
  - Very elegant scheme
  - Extends to any primitive type
- Cons:
  - Hard to embed hierarchical schemes in hardware
  - Complex scenes usually have small polygons and high *depth complexity*
    - Thus most screen regions come down to the single-pixel case

The Z-Buffer Algorithm

- Both BSP trees and Warnock's algorithm were proposed when memory was expensive
  - Example: first 512x512 framebuffer > $50,000!
- Ed Catmull (mid-70s) proposed a radical new approach called *z-buffering*.
- The big idea: resolve visibility *independently at each pixel*

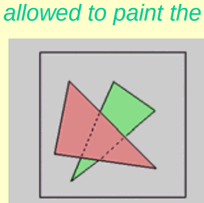
The Z-Buffer Algorithm

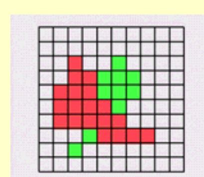
- We know how to rasterize polygons into an image discretized into pixels:



The Z-Buffer Algorithm

- *What happens if multiple primitives occupy the same pixel on the screen? Which is allowed to paint the pixel?*





The Z-Buffer Algorithm

- Idea: retain depth (Z in eye coordinates) through projection transform
  - Use canonical viewing volumes
  - Can transform canonical perspective volume into canonical parallel volume with:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{1+z_{min}} & \frac{-z_{min}}{1+z_{min}} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & (n+f) & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

z-Buffer (*Depth Buffer*)

Conceptually:

$$\underset{(x,y)}{\text{Sort}}\left(\max z_{xy}\right)$$

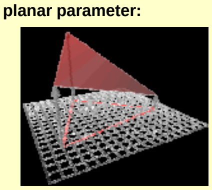
$$\underset{(x,y)}{\text{Sort}}\left(\min z_{xy}\right)$$

The Z-Buffer Algorithm

- Augment framebuffer with *Z-buffer* or *depth buffer* which stores Z value at each pixel
  - At frame beginning initialize all pixel depths to ∞
  - When rasterizing, interpolate depth (Z) across polygon and store in pixel of Z-buffer
  - Suppress writing to a pixel if its Z value is more distant than the Z value already stored there

Interpolating Z

- **Edge equations: Z is just another planar parameter:**
  - ***z = (-D - Ax – By) / C***
  - ***If walking across scanline by (Δx)***
  - ***z_{new} = z – (A/C)(Δx)***
  - Look familiar?
  - Total cost:
    - 1 more parameter to increment in inner loop
    - 3x3 matrix multiply for setup

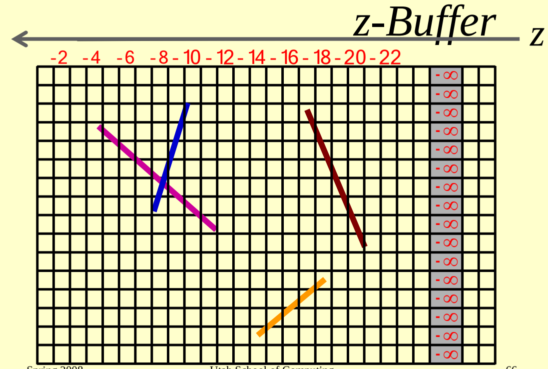


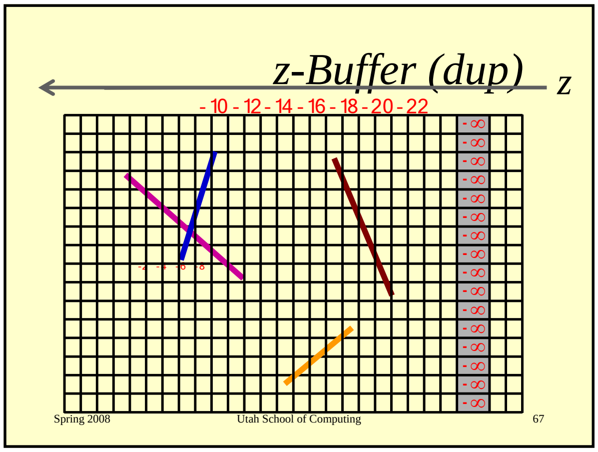
- **Edge walking: just interpolate Z along edges and across spans**

The Z-Buffer Algorithm

- *How much memory does the Z-buffer use?*
- *Does the image rendered depend on the drawing order?*
- *Does the time to render the image depend on the drawing order?*
- *How does Z-buffer load scale with visible polygons? With framebuffer resolution?*

z-Buffer




z-Buffer (dup)
z
- 10 - 12 - 14 - 16 - 18 - 20 - 22
Spring 2008
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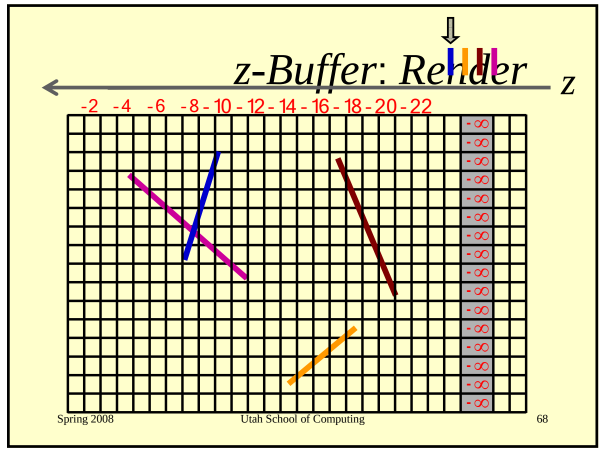
z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
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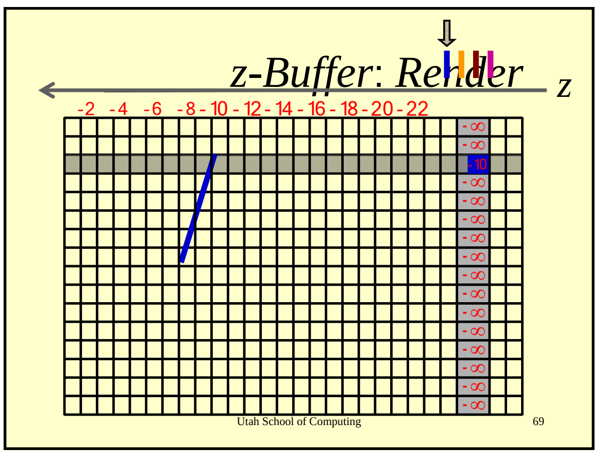
z-Buffer: Render
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-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
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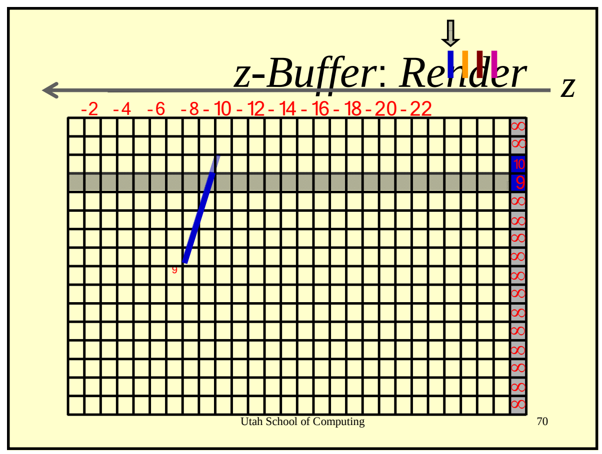
z-Buffer: Render
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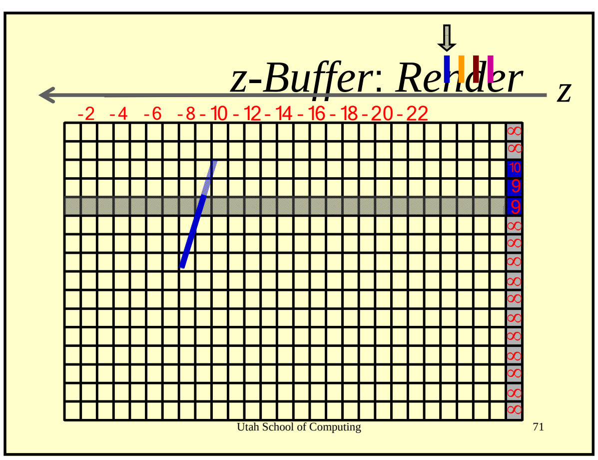
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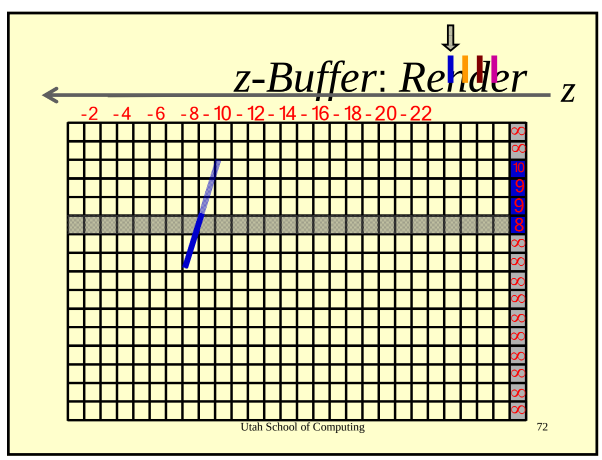
z-Buffer: Render
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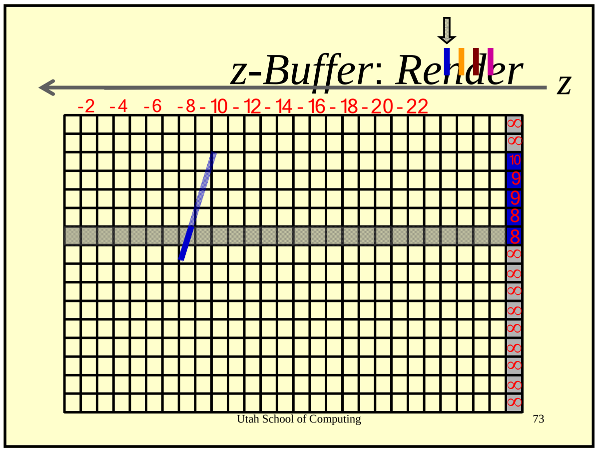
z-Buffer: Render
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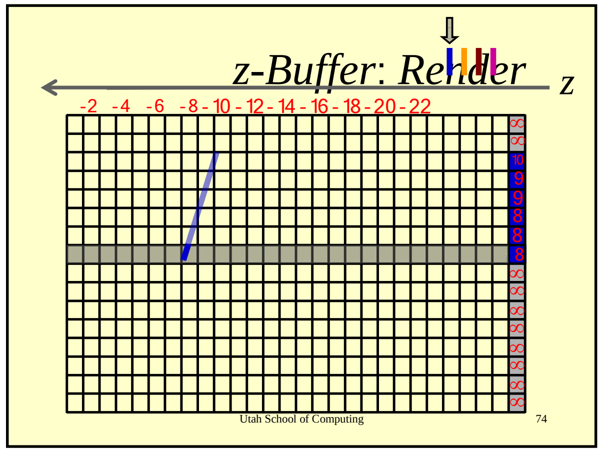
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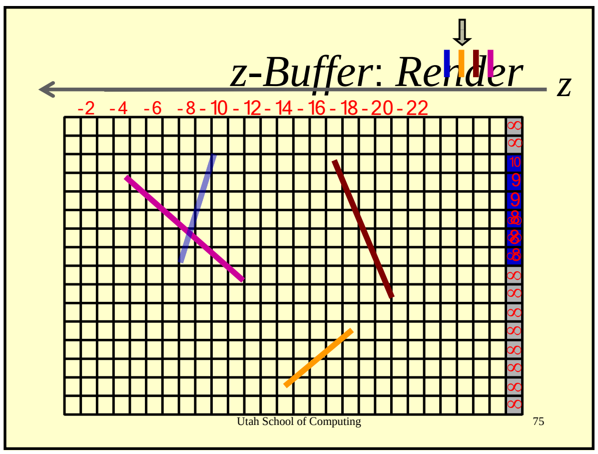
z-Buffer: Render
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-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
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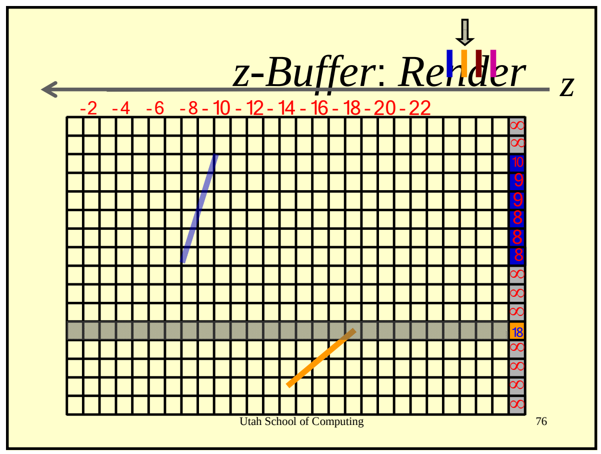
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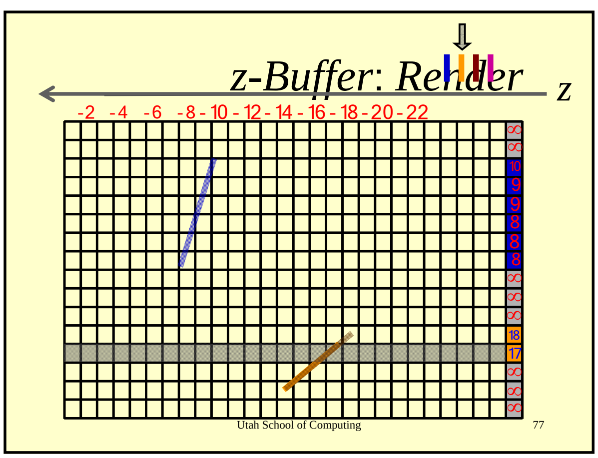
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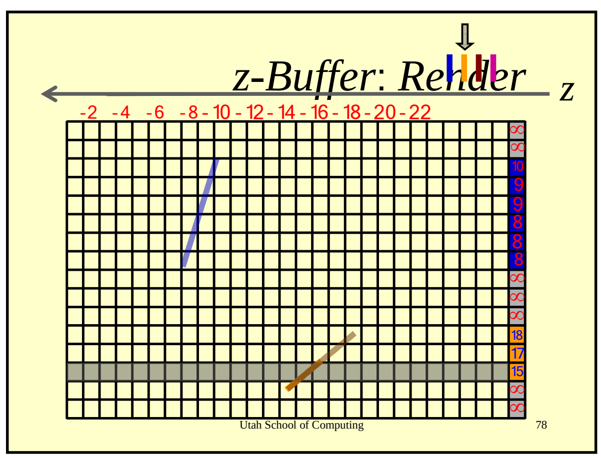
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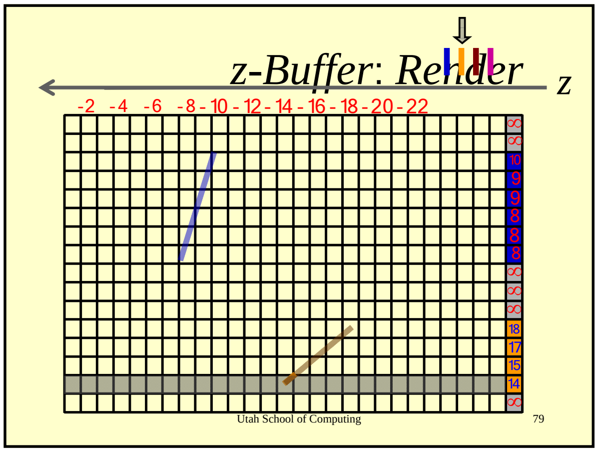
z-Buffer: Render
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-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ ∞ 10 9 9 8 8 8 ∞ ∞ ∞ 18 17 15 14 ∞
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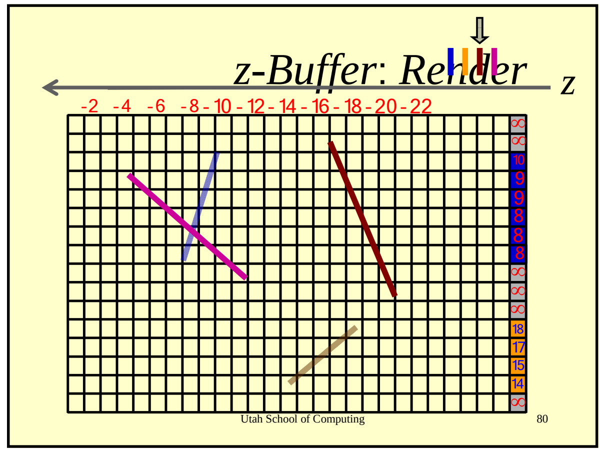
z-Buffer: Render
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∞ ∞ 10 9 9 8 8 8 ∞ ∞ ∞ 18 17 15 14 ∞
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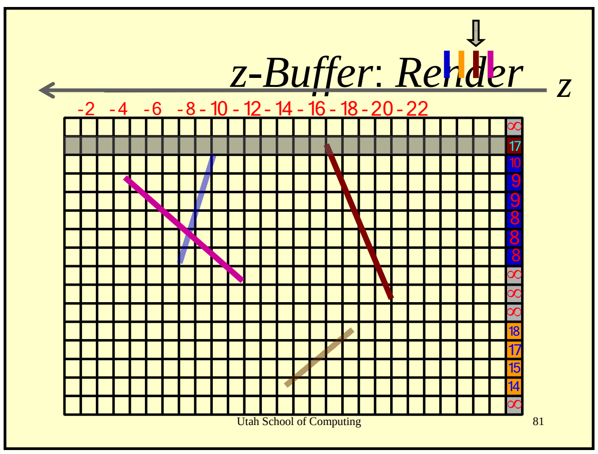
z-Buffer: Render
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∞ 17 10 9 9 8 8 8 ∞ ∞ ∞ 18 17 15 14 ∞
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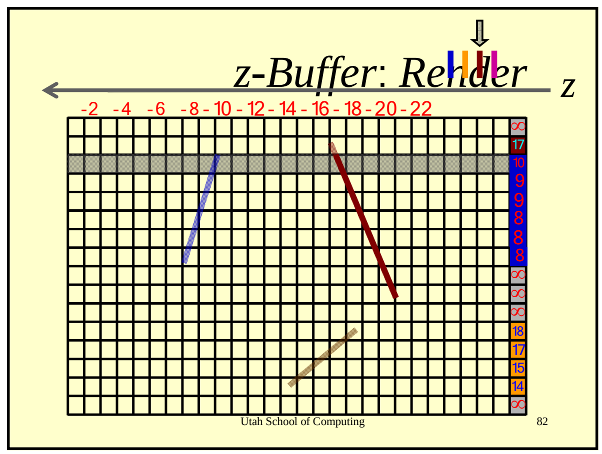
z-Buffer: Render
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-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 9 9 8 8 8 ∞ ∞ ∞ 18 17 15 14 ∞
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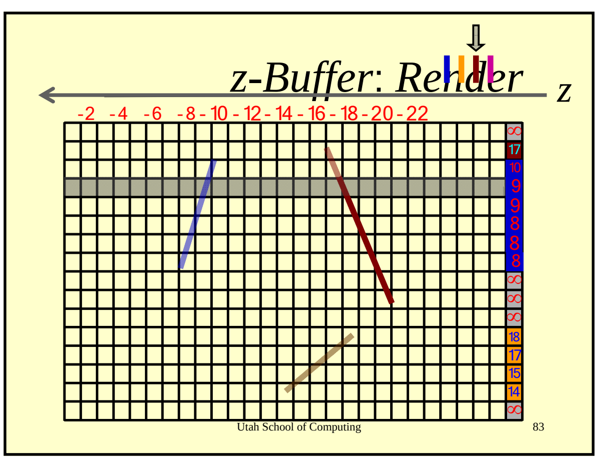
z-Buffer: Render
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-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 9 9 8 8 8 ∞ ∞ ∞ 18 17 15 14 ∞
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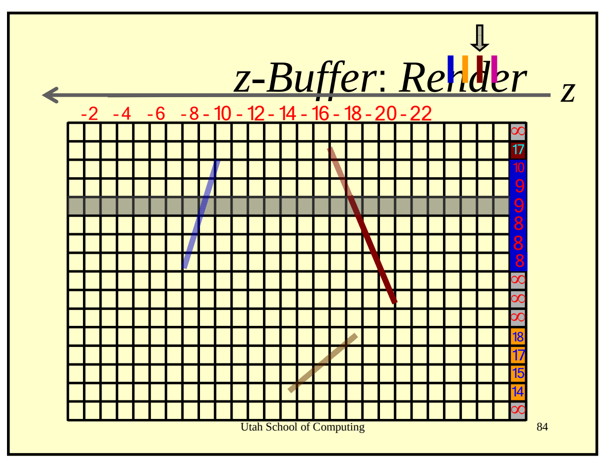
z-Buffer: Render
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-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 9 9 8 8 8 ∞ ∞ ∞ 18 17 15 14 ∞
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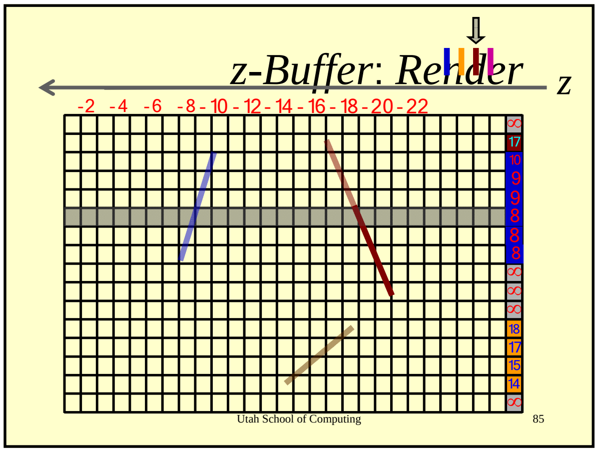
z-Buffer: Render
z
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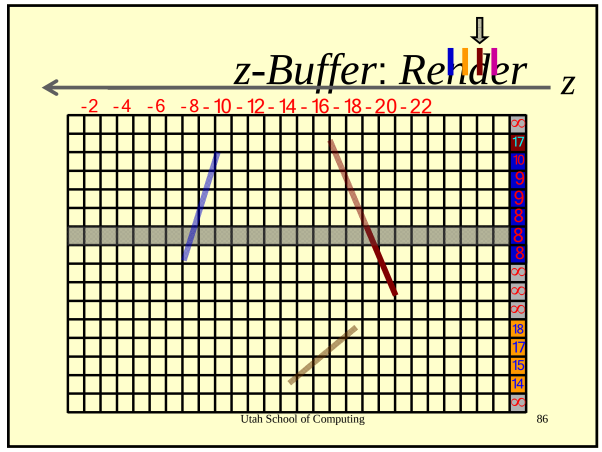
z-Buffer: Render
z
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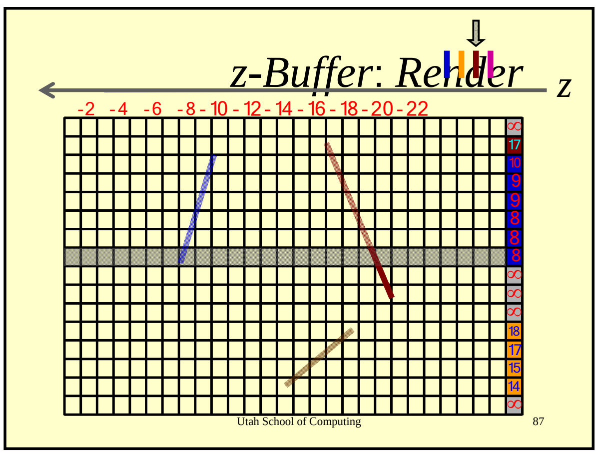
86


z-Buffer: Render
z
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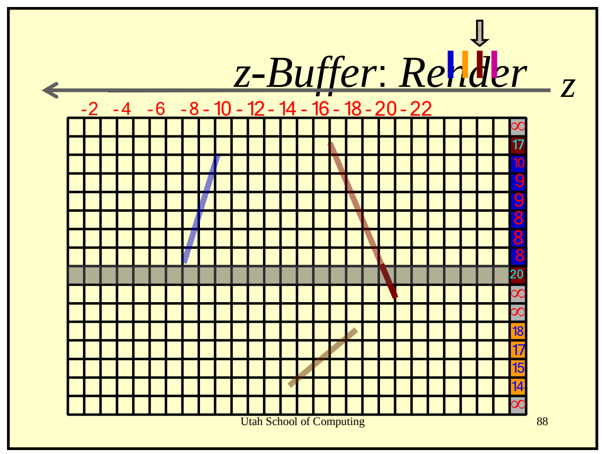
87


z-Buffer: Render
z
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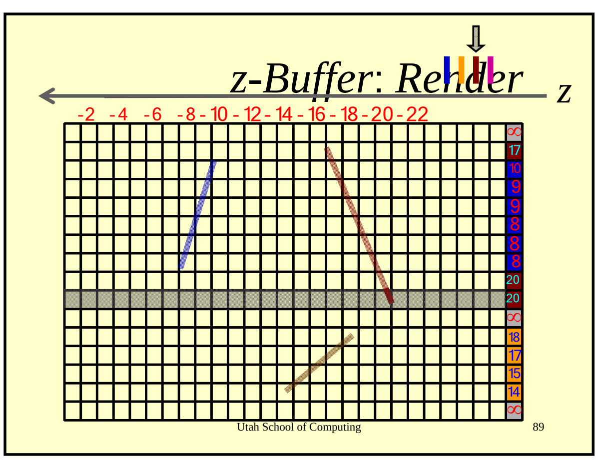
88


z-Buffer: Render
z
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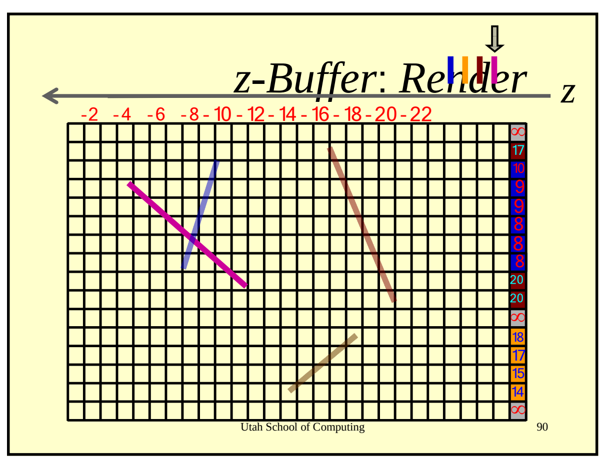
89


z-Buffer: Render
z
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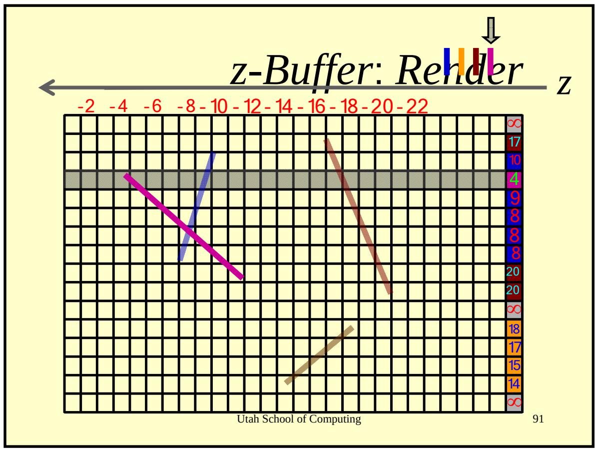
90


z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 4 9 8 8 8 20 20 ∞ 18 17 15 14 ∞
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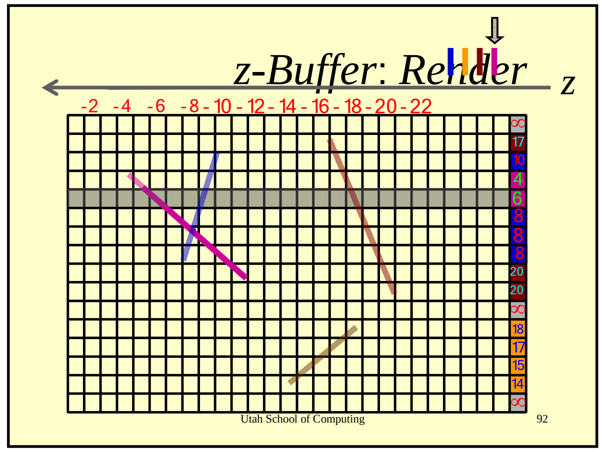
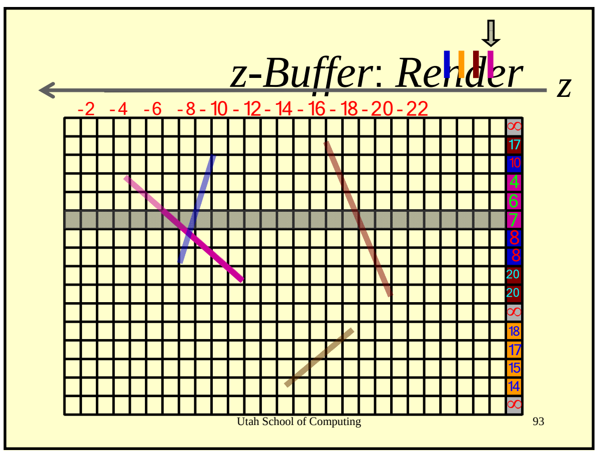
91


z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 4 6 8 8 8 20 20 ∞ 18 17 15 14 ∞
Utah School of Computing
92


z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 4 6 7 8 8 20 20 ∞ 18 17 15 14 ∞
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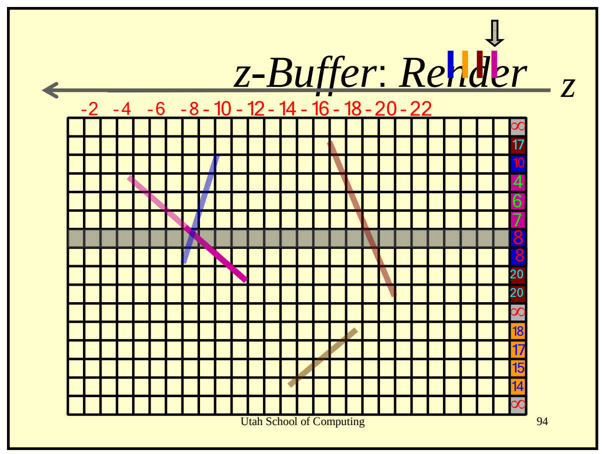
93


z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 4 6 7 8 8 20 20 ∞ 18 17 15 14 ∞
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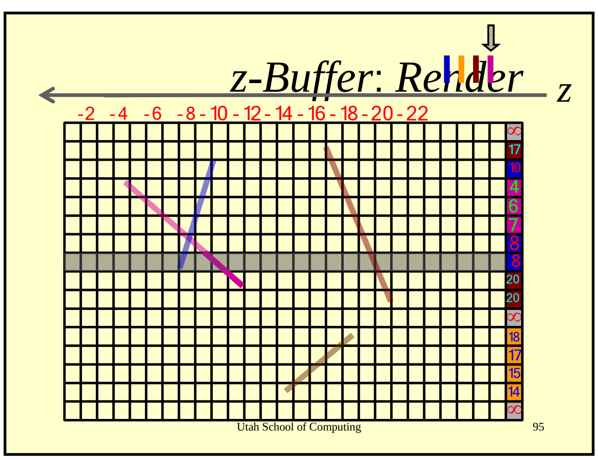
94


z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 4 6 7 8 8 20 20 ∞ 18 17 15 14 ∞
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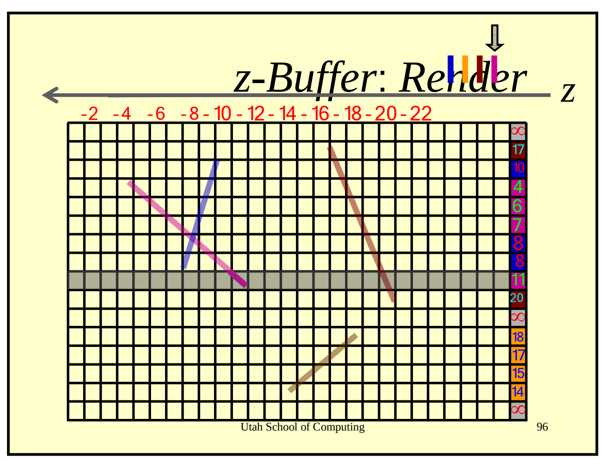
95


z-Buffer: Render
z
-2 -4 -6 -8 - 10 - 12 - 14 - 16 - 18 - 20 - 22
∞ 17 10 4 6 7 8 8 11 20 ∞ 18 17 15 14 ∞
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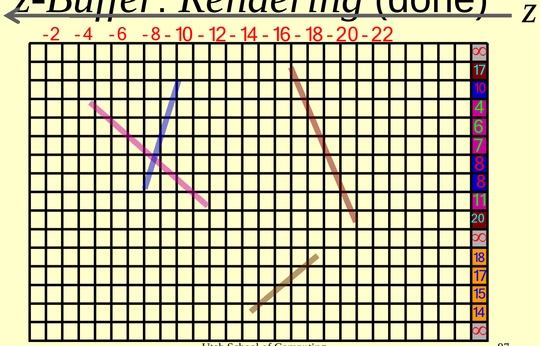
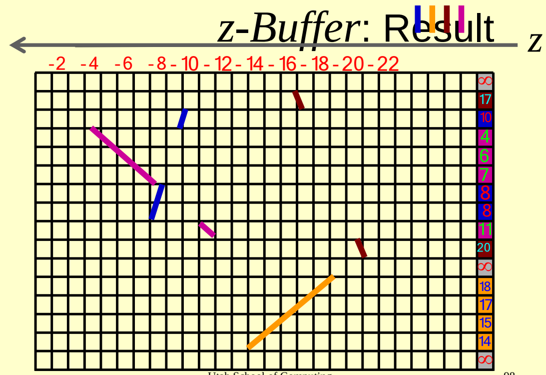
z-Buffer: *Rendering* (done)



z-Buffer: Result



z-Buffer: Pros

- Simple algorithm
- Easy to implement in hardware
- Complexity is order *N*, for polygons
- No polygon processing order required
- Easily handles polygon interpenetration

z-Buffer: Cons

- Memory intensive
- Hard to do antialiasing
- Hard to simulate translucent polygons
- Precision issues (scintillating, worse with perspective projection)

z-Buffer Algorithm

- Initialize buffer
  - Set background *intensity*, *color* <*r*,*g*,*b*>
  - Set *depth* to *max* (*min*) values

z-Buffer Algorithm

- As a *polygon P* is scan converted
  - Calculate depth $z(x,y)$ at each pixel (x,y) being processed
  - Compare $z(x,y)$ with *z-Buffer*(x,y)
  - Replace *z-Buffer*(x,y) with $z(x,y)$ if closer to eye

z-Buffer Algorithm

z

- *Convert* all *polygons*
- Correct image gets generated when *done*
- OpenGL: depth-buffer = z-Buffer

a-Buffer Algorithm

z

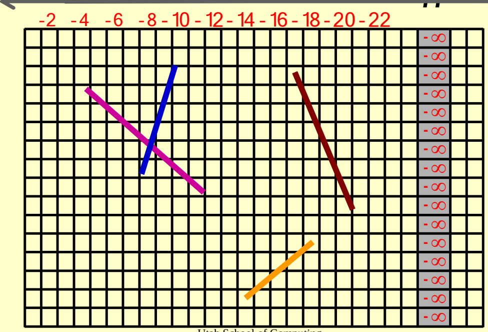
- Generates linked list for each pixel
- Memory of all contributions allows for proper handling of many advanced techniques
- Even *more* memory intensive
- Widely used for high quality rendering

a-Buffer Algorithm: Example

z

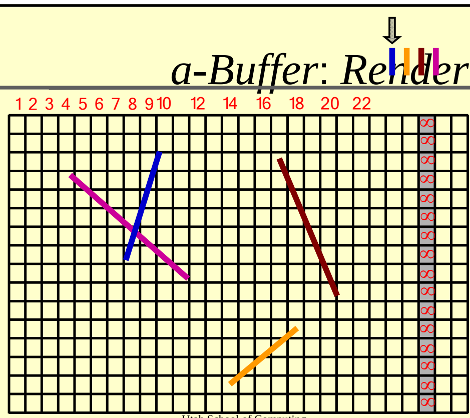
a-Buffer

z



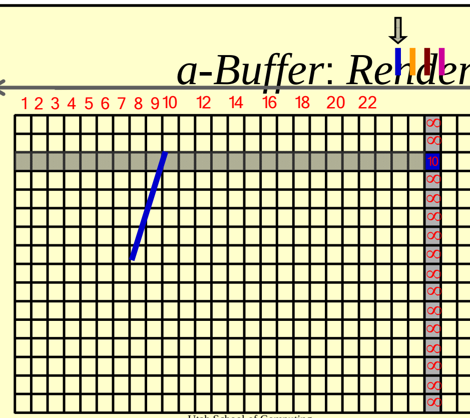
a-Buffer: *Render*

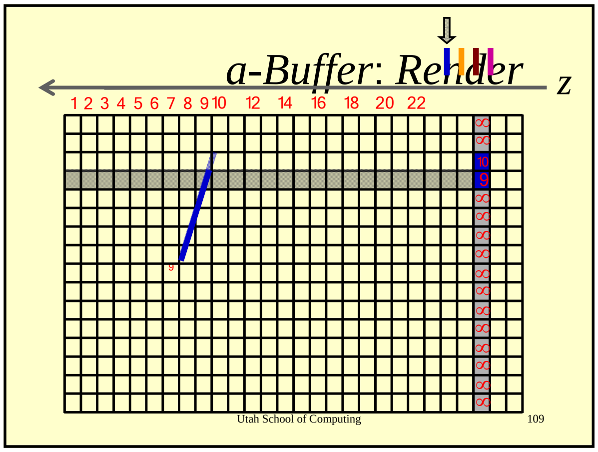
z



a-Buffer: *Render*

z




a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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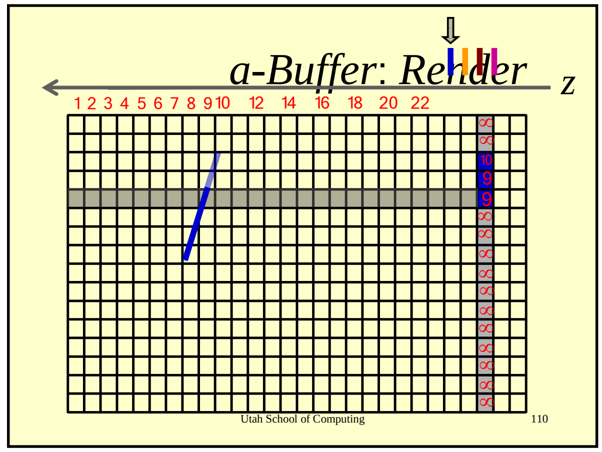
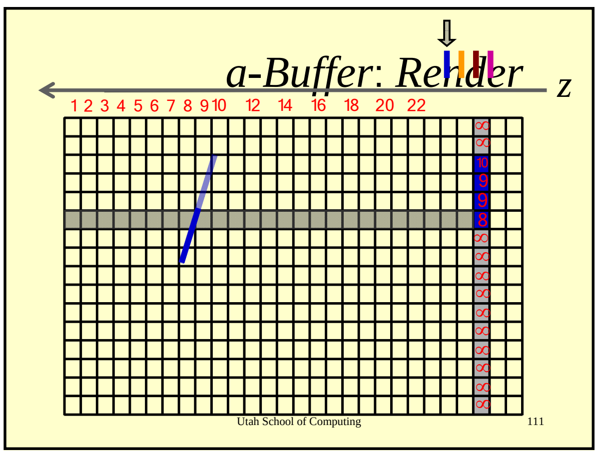

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
110


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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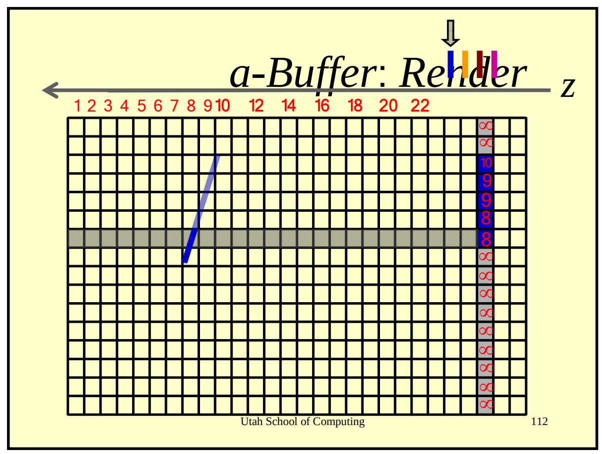
111


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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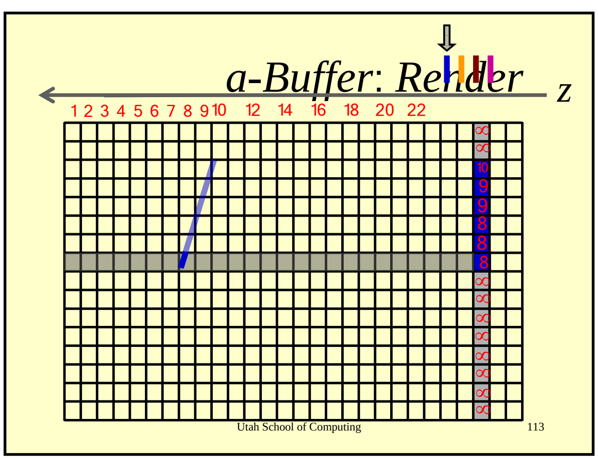
a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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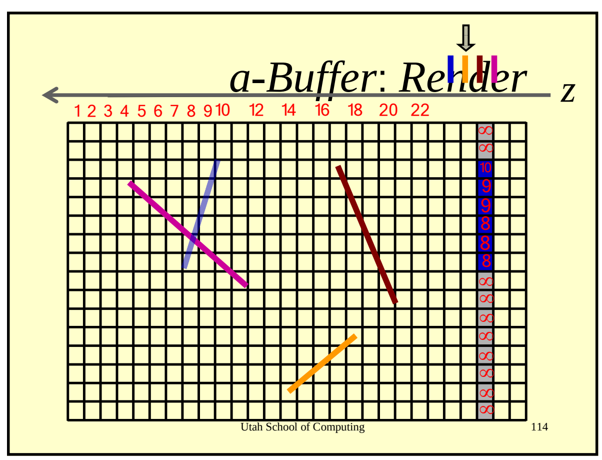
113


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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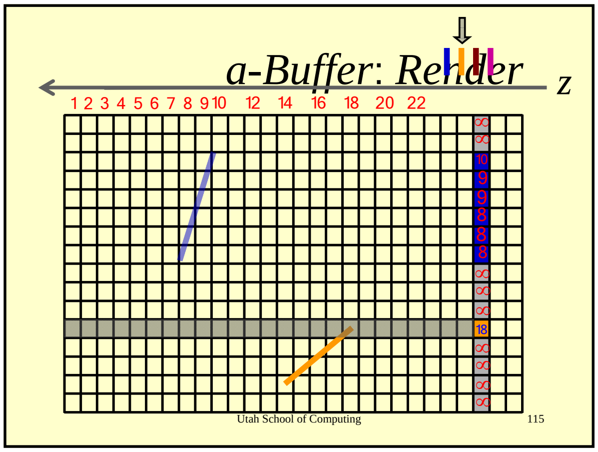
114


a-Buffer: Render
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1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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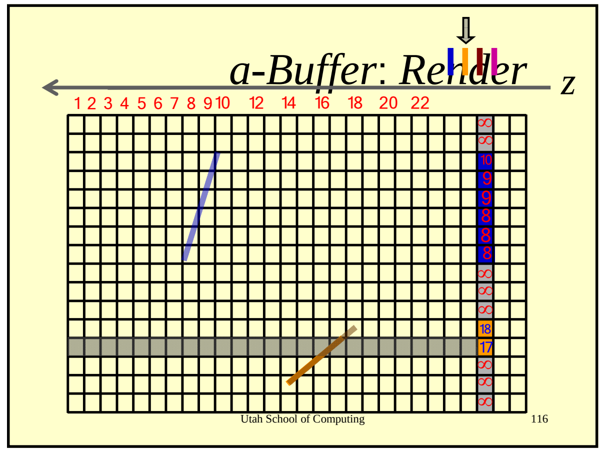
115


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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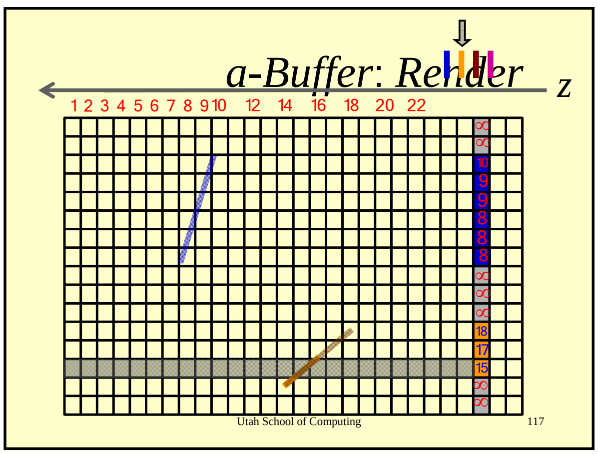
116


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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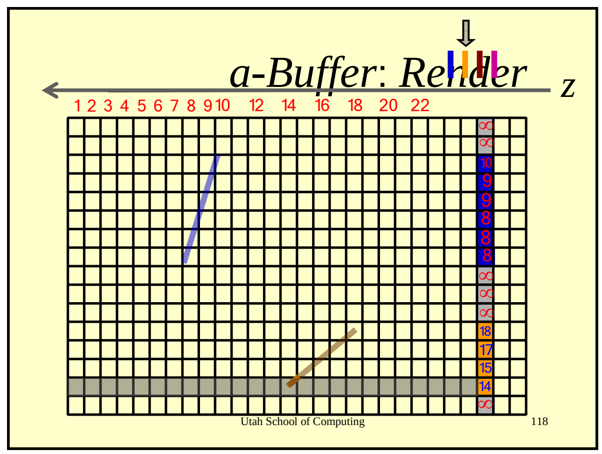
117


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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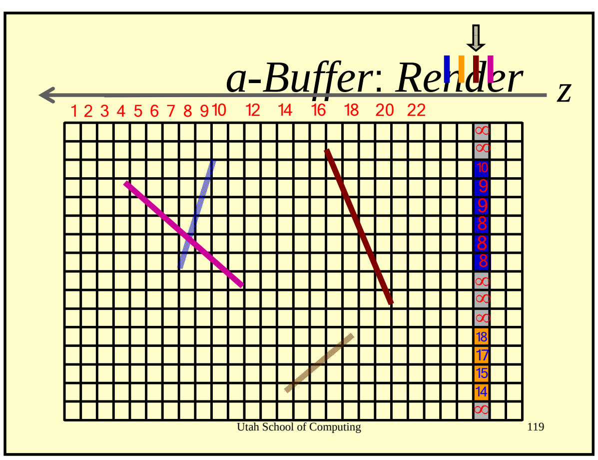
a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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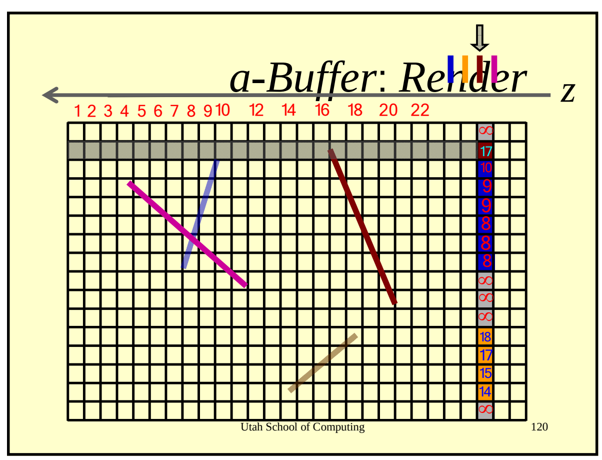
119


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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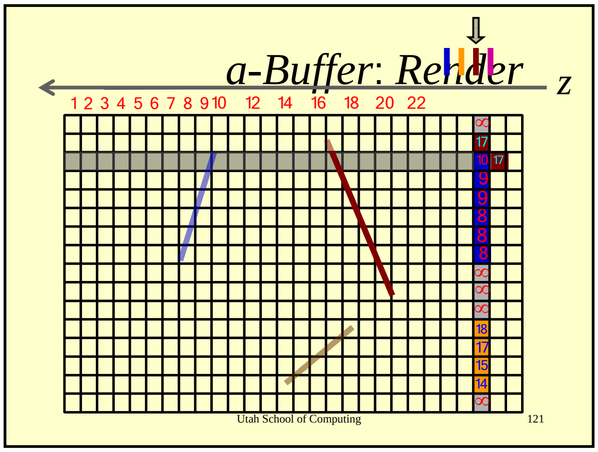
a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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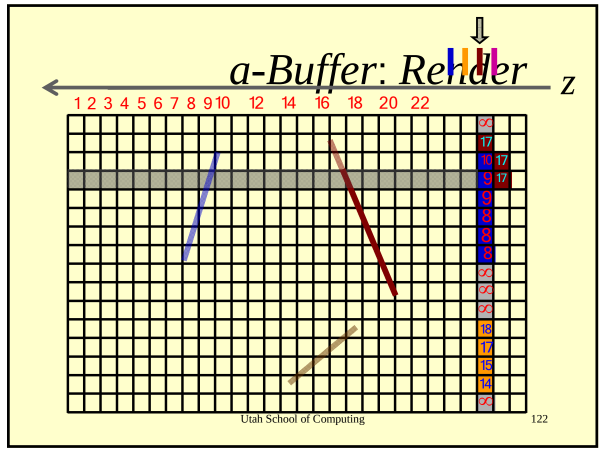
121


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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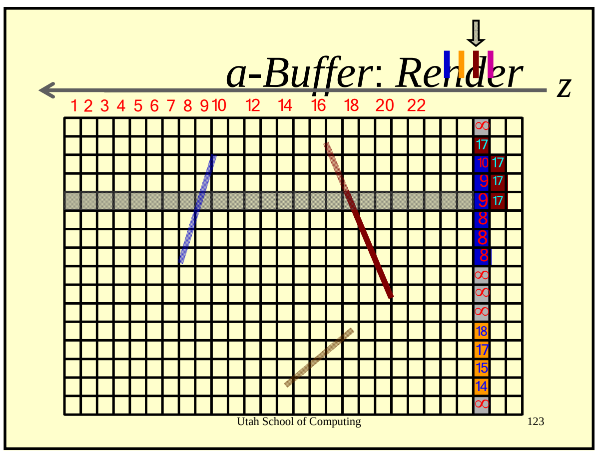
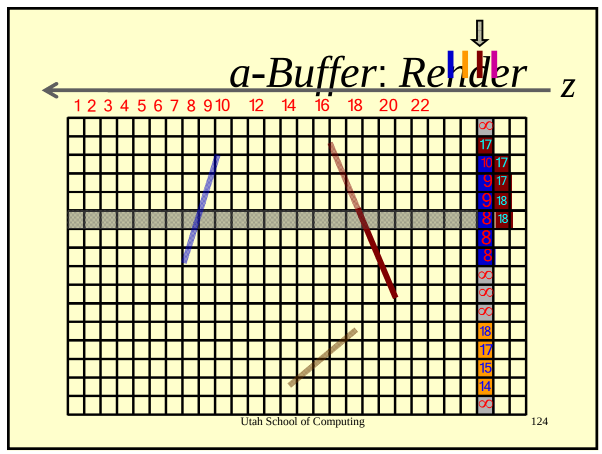
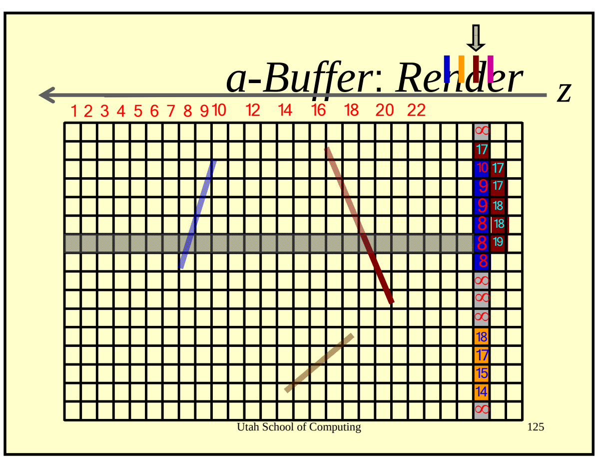
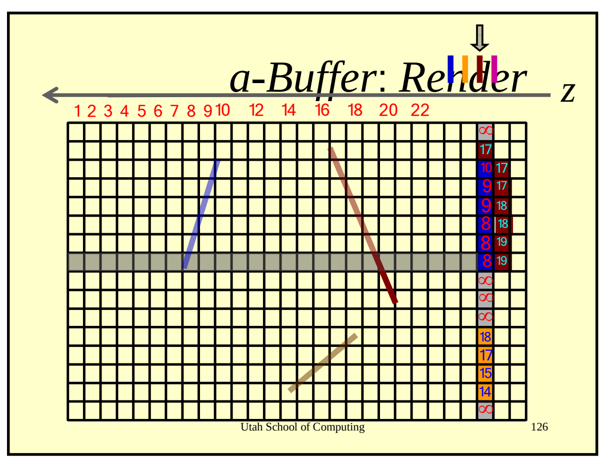
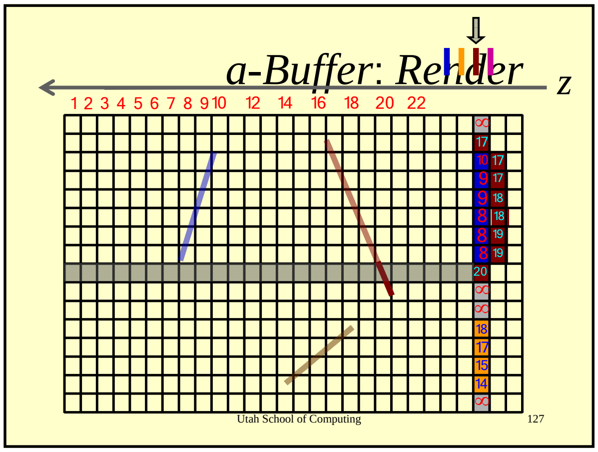
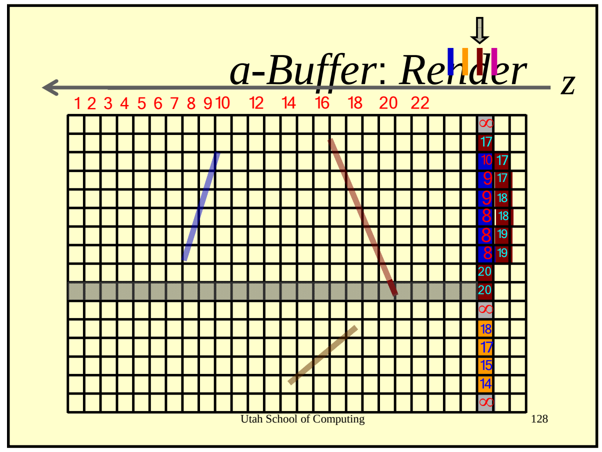

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
123

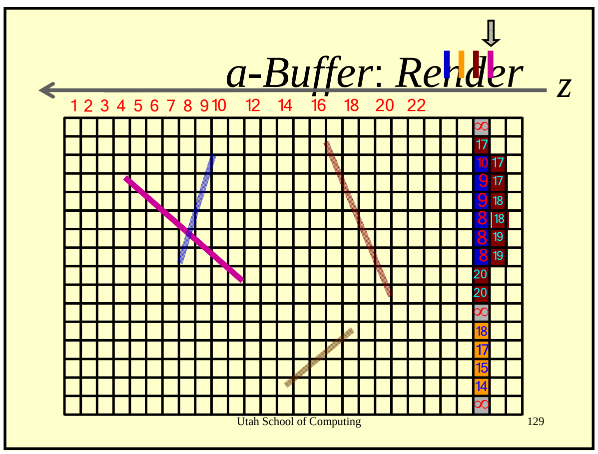

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
124

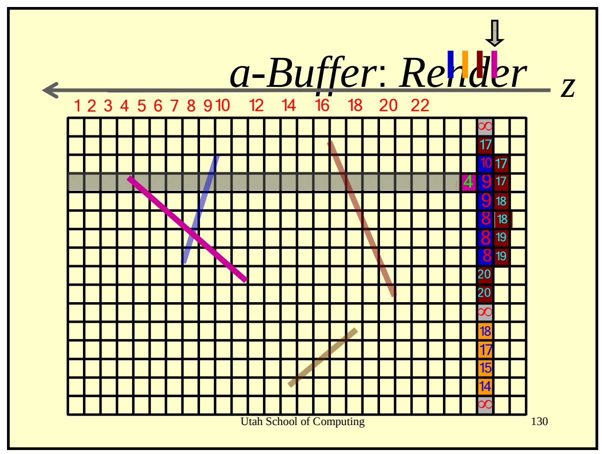

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
125

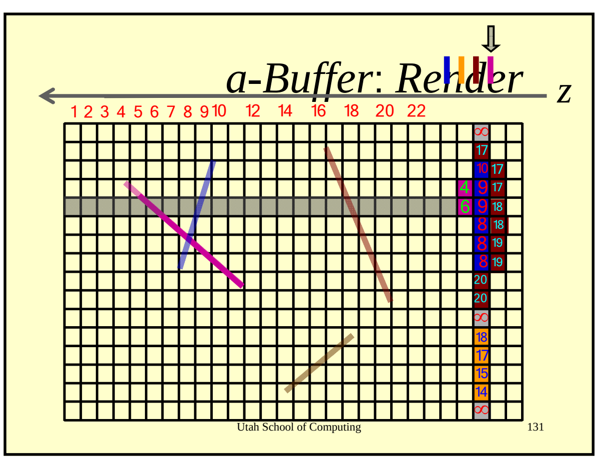

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
126

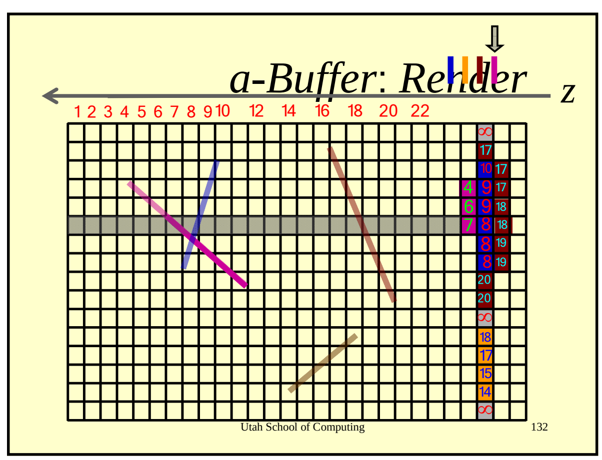

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
127

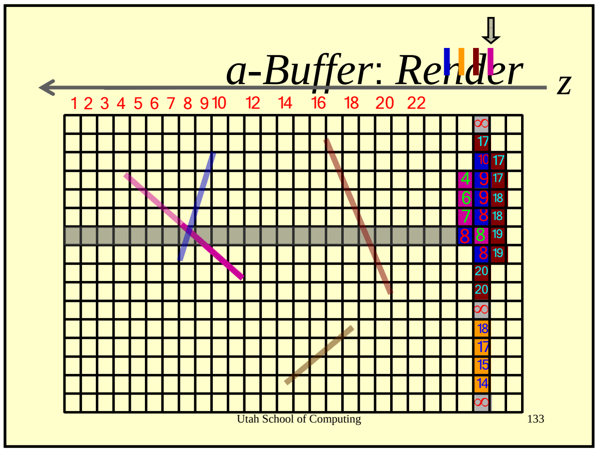

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
128

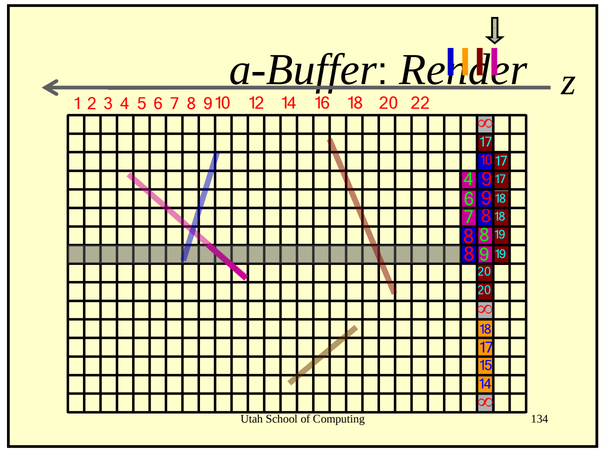

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
129

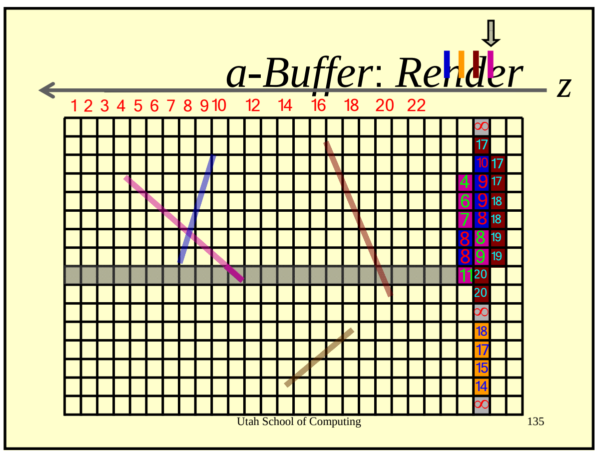

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
130

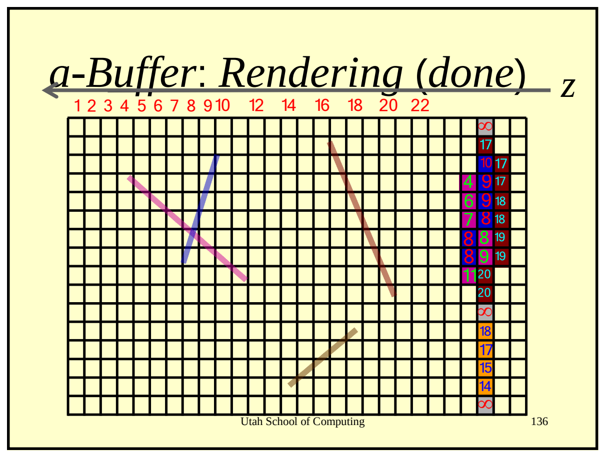

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
131

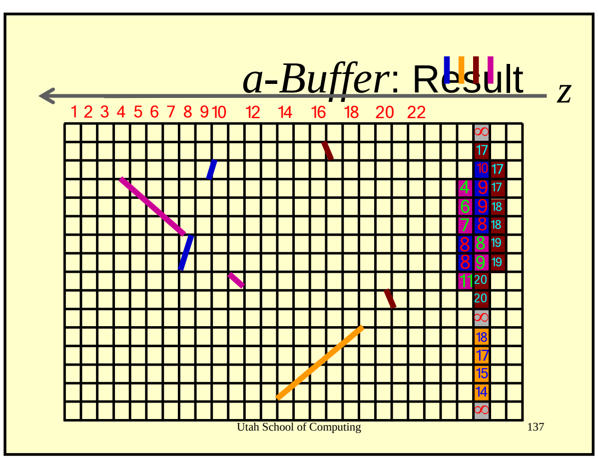

a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
132


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
133


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
134


a-Buffer: Render
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
135


a-Buffer: Rendering (done)
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
Utah School of Computing
136


a-Buffer: Result
z
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22
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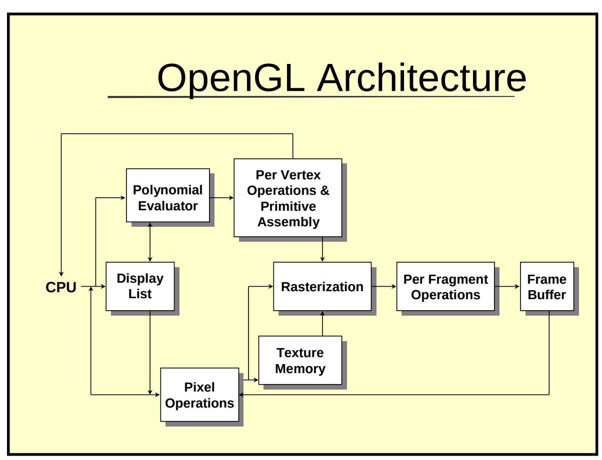
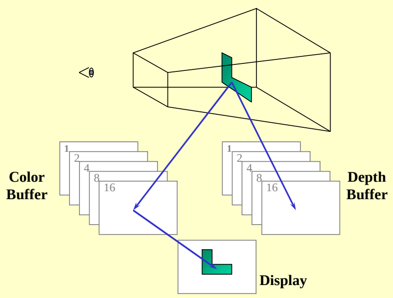
137


OpenGL Architecture
Polynomial Evaluator
Per Vertex Operations & Primitive Assembly
CPU
Display List
Rasterization
Per Fragment Operations
Frame Buffer
Texture Memory
Pixel Operations

Depth Buffering and Hidden Surface Removal



Depth Buffering Using OpenGL

① Request a depth buffer

```
glutInitDisplayMode( GLUT_RGB |
  GLUT_DOUBLE | GLUT_DEPTH );
```

② Enable depth buffering

```
glEnable( GL_DEPTH_TEST );
```

③ Clear color and depth buffers

```
glClear( GL_COLOR_BUFFER_BIT |
  GL_DEPTH_BUFFER_BIT );
```

④ Render scene

⑤ Swap color buffers

An Updated Program Template

```
void main( int argc, char** argv )
{
    glutInit( &argc, argv );
    glutInitDisplayMode( GLUT_RGB |
       GLUT_DOUBLE | GLUT_DEPTH );
    glutCreateWindow( “Tetrahedron”
   );
    init();
    glutIdleFunc( idle );
    glutDisplayFunc( display );
    glutMainLoop();
}
```

An Updated Program Template (cont.)

```
void init( void )
{
   glClearColor( 0.0, 0.0, 1.0, 1.0 );

   glEnable( GL_DEPTH_TEST );
}


void idle( void )
{
   glutPostRedisplay();
}
```

An Updated Program Template (cont.)

```
void drawScene( void )
{
    GLfloat vertices[] = { … };
    GLfloat colors[] = { … };
    glClear( GL_COLOR_BUFFER_BIT |
      GL_DEPTH_BUFFER_BIT );
    glBegin( GL_TRIANGLE_STRIP );
  /* calls to glColor*() and
   glVertex*() */
    glEnd();
    glutSwapBuffers();
}
```

Lecture Set 10

The End

Visible Surface

Determination