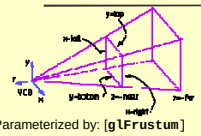
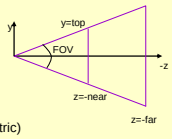
OpenGL Projection Tutorial

View Frustum





Parameterized by: [**glFrustum**]
- left,right,top,bottom (generally symmetric)
- near,far

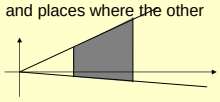
Or, when symmetric, by: [**gluPerspective**]
- *Field of view* (FOV), *aspect ratio*
- near,far
- Aspect ratio is the x/y ratio of the final displayed image. Common values:
  - 4/3 for TV & old movies; 1.66 for cartoons & European movies; 16/9 for American movies & HDTV; 2.35 for epic movies

$$\text{aspect ratio} = \frac{right - left}{top - bottom} = \frac{right}{top}$$

$$\tan(FOV/2) = \frac{top}{near}$$

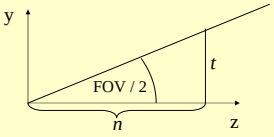
OpenGL

- `gluPerspective(…)`
  - Field of view in the *y* direction, *FOV*, (vertical field-of-view)
  - Aspect ratio, ***a***, should match window aspect ratio
  - Near and far clipping planes, *n* and *f*
  - Defines a symmetric view volume
- `glFrustum(…)`
  - Give the near and far clip plane, and places where the other clip planes cross the near plane
  - Defines the general case
  - Used for stereo viewing, mostly



`gluPerspective` to `glFrustum`

- As noted previously, `glu` functions don't add basic functionality, they are just more convenient
  - So how does `gluPerspective` convert to `glFrustum`?
  - Symmetric, so only need *t* and *l*

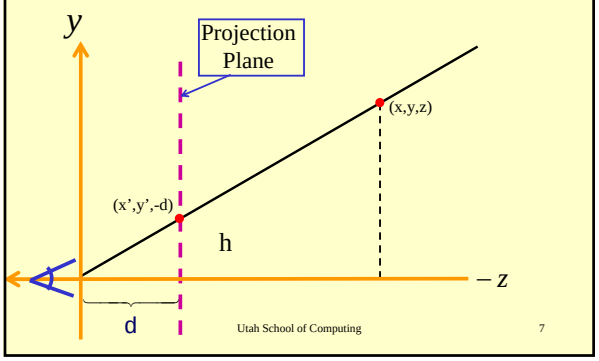


?

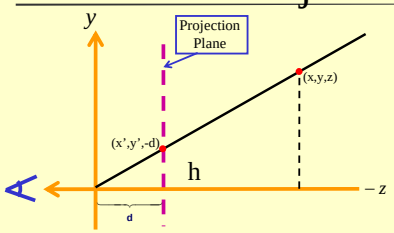
Demo Projection Tutor

Viewing System PDFs

3D Projection



3D Projection



$$\frac{X}{Z} = \frac{X'}{d} \Rightarrow X' = \frac{d * X}{Z} \qquad \frac{Y}{Z} = \frac{Y'}{d} \Rightarrow Y' = \frac{d * Y}{Z}$$

3D Projection

$$\text{Proj} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix} = \begin{bmatrix} \frac{x*d}{z} \\ \frac{y*d}{z} \\ d \\ 1 \end{bmatrix}$$

3D Projection

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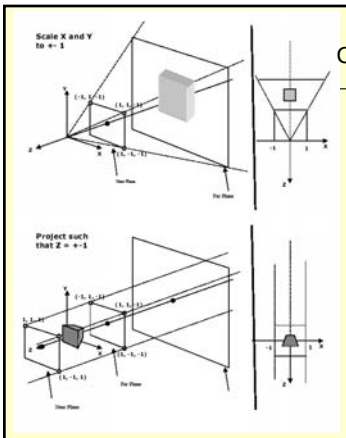
How many vanishing points?

PDF of OpenGL projection



Canonical View Volume

Canonical to Window

- Canonical Viewing Volume (what is it?)
- To Window

$$\mathbf{M}_{window} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{sys} = \mathbf{M}_{window}\mathbf{M}_{persp}\mathbf{M}_{view}$$

Complete Perspective Projection

- After applying the perspective matrix, we map the orthographic view volume to the canonical view volume:

$$\mathbf{M}_{persp} = \mathbf{M}_O\mathbf{M}_P = \begin{bmatrix} \frac{2}{(r-l)} & 0 & 0 & \frac{(r+l)}{(r-l)} \\ 0 & \frac{2}{(t-b)} & 0 & \frac{(t+b)}{(t-b)} \\ 0 & 0 & \frac{2}{(n-f)} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & (n+f) & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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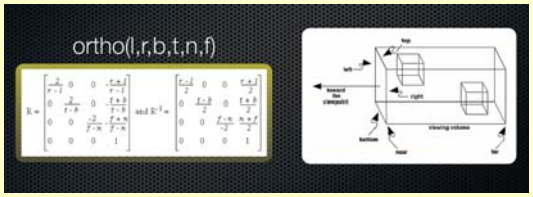
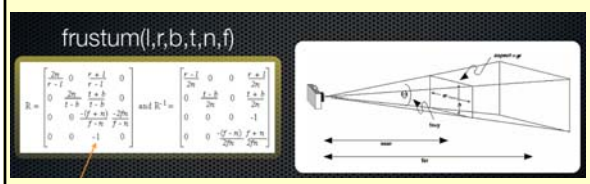
glViewport() glufrustum() gluLookAt()

Full OpenGL Ortho Projection



Full OpenGL Perspective Proj



GL Matrix Example

```
// Clear screen
glClear(GL_COLOR_BUFFER_BIT,GL_DEPTH_BUFFER_BIT);

// Set up projection
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
gluPerspective(fov,aspect,nearclip,farclip);

// Set up camera view
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(eye.x,eye.y,eye.z,target.x,target.y,target.z,0,1,0);

// Draw all objects
for(each object) {
      glPushMatrix();
      glTranslatef(pos[i].x,pos[i].y,pos[i].z);
      glRotatef(axis[i].x,axis[i].y,axis[i].z,angle[i]);
      Model[i]->Draw();
      glPopMatrix();
}

// Finish
glFlush();
glSwapBuffers();
```