What is "*Perspective*?"

- A mechanism for portraying $3D$ in $2D$
- "*True Perspective*" corresponds to projection onto a plane
- "*True Perspective*" corresponds to an ideal camera image

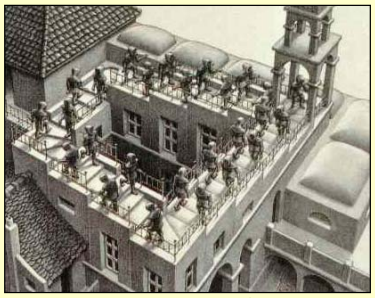
Perspective (Mural) Games

M C Esher, *Another World II* (1947)

Perspective

M.C. Escher, Ascending and Descending (1960)

M. C. Esher



M.C. Escher, Ascending and Descending (1960)

M. C. Esher

- Perspective is "local"
- Perspective consistency is not "transitive"
- Nonplanar (hyperbolic) projection

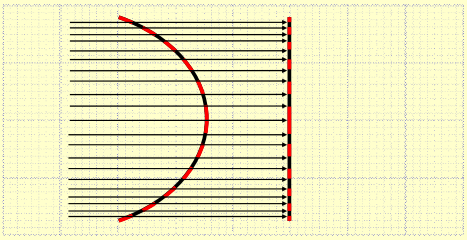
Nonplanar (Hyperbolic) Projection

M C Esher, *Heaven and Hell*

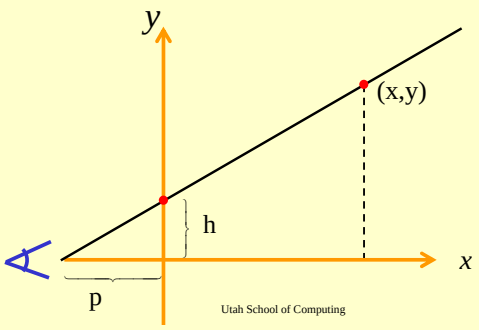
Nonplanar (*Hyperbolic)Projection*

M C Esher, *Heaven and Hell*

Curvilinear Projection



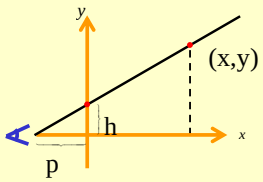
"True" Perspective in 2D



"True" Perspective in 2*D*

$$\frac{h}{p} = \frac{y}{x+p}$$

$$h = \frac{py}{x+p}$$



"True" Perspective in 2*D*

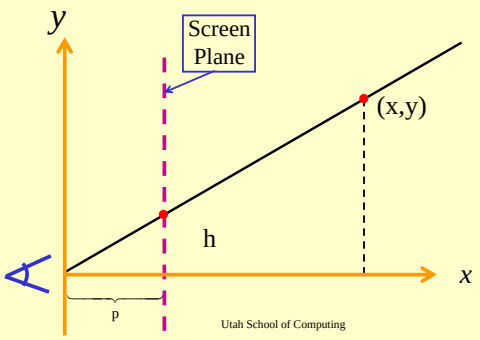
$$\begin{bmatrix} ? \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \end{bmatrix}$$

"True" Perspective in 2*D*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/p & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ x/p + 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ \frac{x+p}{p} \end{bmatrix} = \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \end{bmatrix}$$

Geometry Same for Eye at Origin



What Happens to Special Points?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/p & 0 & 1 \end{bmatrix} \begin{bmatrix} -p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -p \\ 0 \\ 0 \end{bmatrix}$$

What is this point??

Let's Look at a Limit

Observe,

$$\lim_{n \to \infty} \begin{bmatrix} 1 \\ 0 \\ \left(\frac{1}{n}\right) \end{bmatrix} = \begin{bmatrix} n \\ 0 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} n \\ 0 \end{bmatrix}$$

We see that $\begin{bmatrix} n \\ 0 \end{bmatrix} \Leftrightarrow +\infty \text{ on } x\text{-axis}$

Where does Eye Point Go?

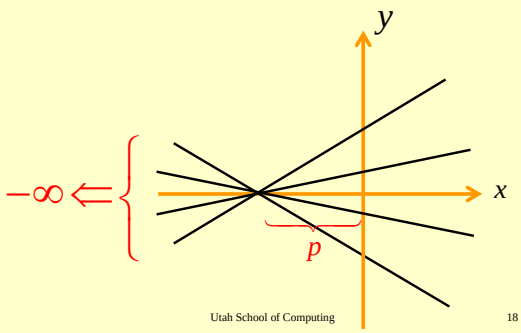
- It gets sent to -∞ on *x-axis*
- Where does +∞ go on *x-axis* go?

What happens to +∞?

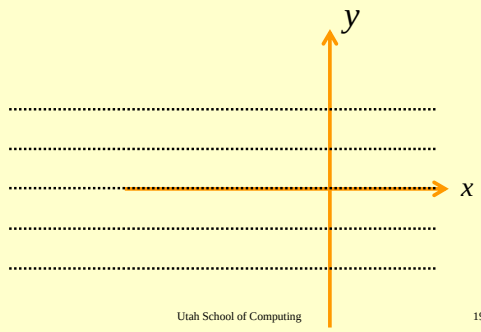
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/p & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1/p \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} p \\ 0 \end{bmatrix}$$

It comes back to virtual eye point!

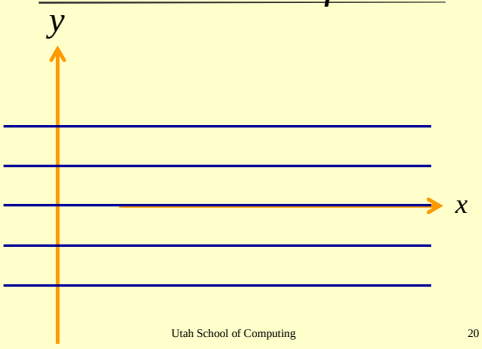
What Does This Mean?



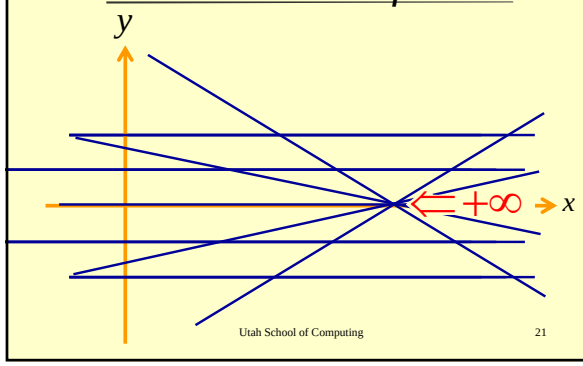
"Pencil of Lines" Becomes Parallel



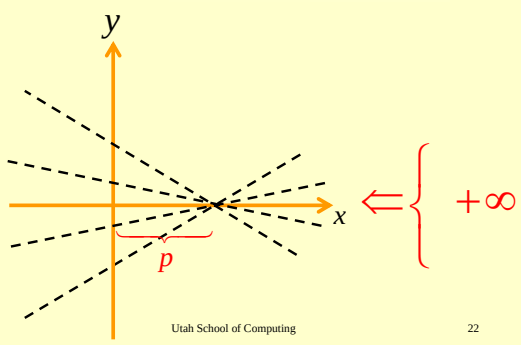
Parallel Lines Become a "*Pencil of Lines*" !



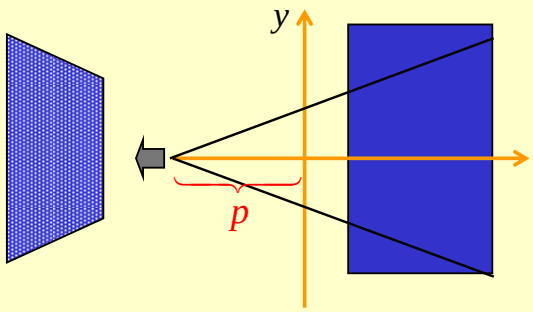
Parallel Lines Become "*Pencil of Lines*" !



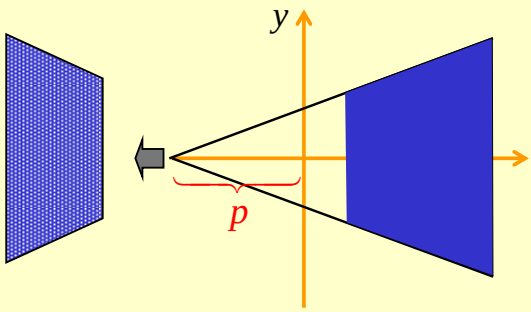
What Does This Mean?



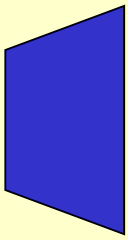
"True" Perspective in *2D*



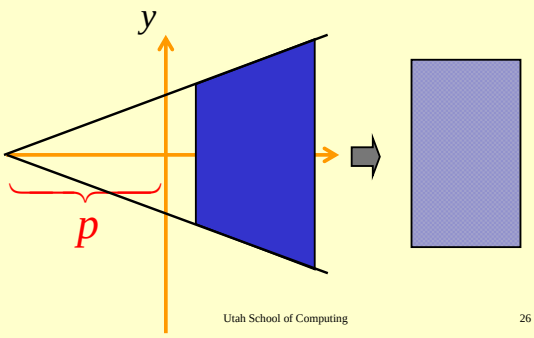
"True" Perspective in *2D*



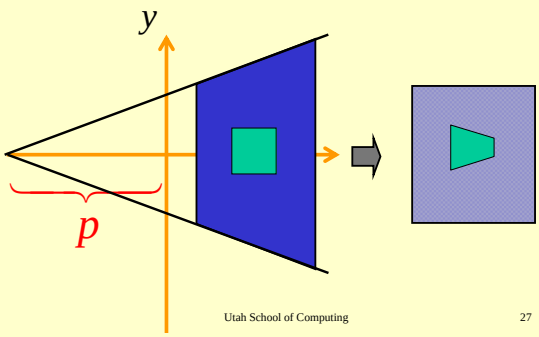
Viewing *Frustum*



"True" Perspective in *2D*



"True" Perspective in *2D*



What happens for large *p*?"

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/p & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\lim_{p \to \infty} \frac{1}{p} = 0$$

Projection Becomes Orthogonal: "Right Thing Happens"