Lecture Set 3

# *Bresenham Circles*

CS5600 *Intro to Computer Graphics*

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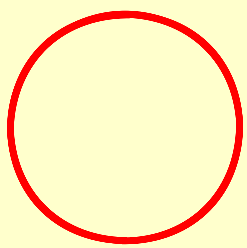
Spring 2013



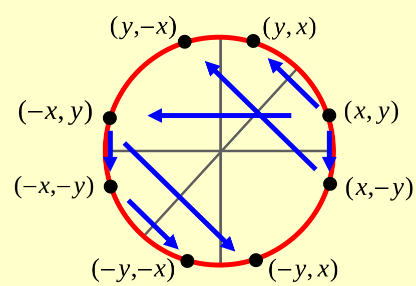
## More Raster Line Issues

- Fat lines with multiple pixel width
- Symmetric lines
- End point geometry – how should it look?
- Generating curves, e.g., circles, etc.
- Jaggies, staircase effect, aliasing...

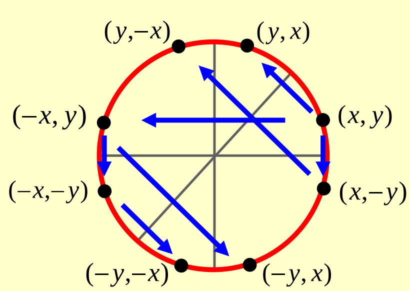
## Generating Circles



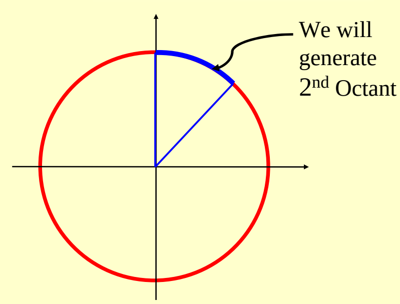
## Exploit 8-Point Symmetry




## Once More: 8-Pt Symmetry




## Only 1 Octant Needed



## Generating pt $(x,y)$ gives

the following 8 pts by symmetry:

$\{(x,y), (-x,y), (-x,-y), (x,-y),$
$(y,x), (-y,x), (-y,-x), (y,-x)\}$

## $2^{nd}$ Octant Is a Good Arc

- It is a function in this domain
  - single-valued
  - no vertical tangents: $|slope| \le 1$
- Lends itself to Bresenham
  - only need consider $E$ or $SE$

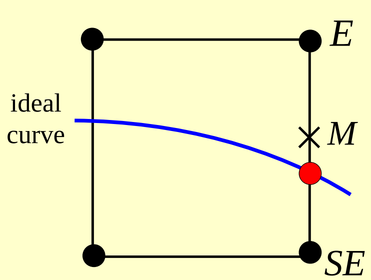
## Implicit Eq's for Circle

- Let $F(x,y) = x^2 + y^2 - r^2$
- For $(x,y)$ on the circle, $F(x,y) = 0$
- So, $F(x,y) > 0 \Rightarrow (x,y)$ *Outside*
- And, $F(x,y) < 0 \Rightarrow (x,y)$ *Inside*

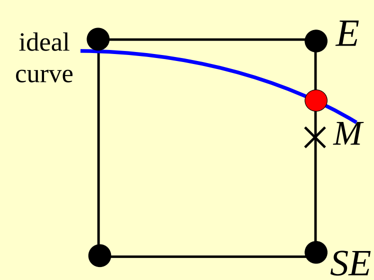
## Choose $E$ or $SE$

- Function is $x^2 + y^2 - r^2 = 0$
- So, $F(M) \ge 0 \Rightarrow SE$
- And, $F(M) < 0 \Rightarrow E$

## $F(M) \ge 0 \Rightarrow SE$




## $F(M) < 0 \Rightarrow E$

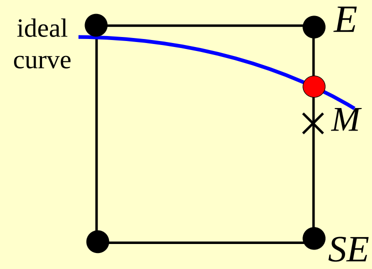


## Decision Variable $d$

Again, we let,

$d = F(M)$

## Look at Case 1: $E$




## $d_{old} < 0 \Rightarrow E$

$$d_{old} = F(x_p + 1, y_p - \tfrac{1}{2})$$
$$= (x_p + 1)^2 + (y_p - \tfrac{1}{2})^2 - r^2$$

## $d_{old} < 0 \Rightarrow E$

$$d_{new} = F(x_p + 2, y_p - \tfrac{1}{2})$$
$$= (x_p + 2)^2 + (y_p - \tfrac{1}{2})^2 - r^2$$

## $d_{old} < 0 \Rightarrow E$

$$d_{new} = d_{old} + (2x_p + 3)$$

Since, $d_{new} - d_{old}$

$$(x_p + 2)^2 - (x_p + 1)^2 = (x_p^2 + 4x_p + 4) - (x_p^2 + 2x_p + 1)$$
$$= 2x_p + 3$$

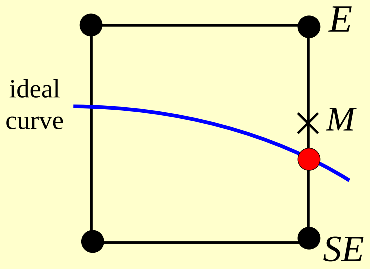
## $d_{old} < 0 \Rightarrow E$

$$d_{new} = d_{old} + \Delta_E \;,$$

where,

$$\boxed{\Delta_E = 2x_p + 3}$$

## Look at Case 2: *SE*




## $d_{old} \geq 0 \Rightarrow SE$

$$d_{new} = F(x_p + 2,\ y_p - 3/2)$$

$$= (x_p + 2)^2 + (y_p - 3/2)^2 - r^2$$

$$d_{new} = d_{old} + (2x_p - 2y_p + 5)$$

Because,…, straightforward manipulation

## $d_{old} \geq 0 \Rightarrow SE$

$$d_{new} - d_{old} =$$

$$(x_p+2)^2 + (y_p - \frac{3}{2})^2 - r^2 - \left[(x_p+1)^2 + (y_p - \frac{1}{2})^2 - r^2\right]$$

$$= (2x_p + 3) + y_p^2 - 3y_p + \frac{9}{4} - \left[y_p^2 - y_p + \frac{1}{4}\right]$$

## $d_{old} \geq 0 \Rightarrow SE$

$$d_{new} - d_{old} =$$

$$(x_p+2)^2 + (y_p - \frac{3}{2})^2 - r^2 - \left[(x_p+1)^2 + (y_p - \frac{1}{2})^2 - r^2\right]$$

$$= (2x_p + 3) + y_p^2 - 3y_p + \frac{9}{4} - \left[y_p^2 - y_p + \frac{1}{4}\right]$$

## $d_{old} \geq 0 \Rightarrow SE$

$$d_{new} - d_{old} =$$

$$(x_p+2)^2 + (y_p - \frac{3}{2})^2 - r^2 - \left[(x_p+1)^2 + (y_p - \frac{1}{2})^2 - r^2\right]$$

$$= (2x_p + 3) + y_p^2 - 3y_p + \frac{9}{4} - \left[y_p^2 - y_p + \frac{1}{4}\right]$$

## $d_{old} \geq 0 \Rightarrow SE$

$$d_{new} - d_{old} =$$

$$\underbrace{(2x_p + 3)}_{\text{From } \Delta_E \text{ calculation}} + \underbrace{(-3y_p + \frac{9}{4})}_{\text{From } new \text{ y-coordinate}} - \underbrace{(-y_p + \frac{1}{4})}_{\text{From } old \text{ y-coordinate}}$$

$$d_{old} \geq 0 \implies SE$$

I.e.,

$$d_{new} = d_{old} + (2x_p - 2y_p + 5)$$
$$= d_{old} + \Delta_{SE}$$

$$\boxed{\Delta_{SE} = 2x_p - 2y_p + 5}$$

## Note: Δ´s Not Constant

$\Delta_E$ *and* $\Delta_{SE}$

depend on values of $x_p$ and $y_p$

## Summary

- Δ´s are no longer constant over entire line
- Algorithm structure is *exactly* the same
- Major difference from the line algorithm
  - Δ is re-evaluated at each step
  - Requires *real* arithmetic

## Initial Condition

- Let *r* be an integer. Start at $(0, r)$
- Next midpoint *M* lies at $(1, r - \frac{1}{2})$
- So, $F(1, r - \frac{1}{2}) = 1 + (r^2 - r - \frac{1}{4}) - r^2$

$$\boxed{= \frac{5}{4} - r}$$

## Ellipses

- Evaluation is analogous
- Structure is same
- Have to work out the Δ´s

## Getting to Integers

- Note the previous algorithm involves *real* arithmetic
- Can we modify the algorithm to use integer arithmetic?

## Integer Circle Algorithm

- Define a shift decision variable

$$h = d - \frac{1}{4}$$

- In the code, plug in $d = h + \frac{1}{4}$

## Integer Circle Algorithm

- Now, the initialization is $h = 1 - r$
- So the initial value becomes

$$F(1, r - \frac{1}{2}) - \frac{1}{4} = (\frac{5}{4} - r) - \frac{1}{4}$$
$$= 1 - r$$

## Integer Circle Algorithm

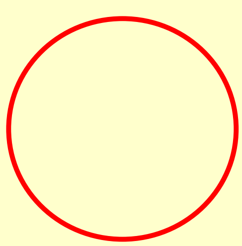
- Then, $d < 0$ becomes $h < -\frac{1}{4}$
- Since $h$ an integer

$$\boxed{h < -\frac{1}{4} \quad \Leftrightarrow \quad h < 0}$$

## Integer Circle Algorithm

- But, $h$ begins as an integer
- And, $h$ gets incremented by integer
- Hence, we have an integer circle algorithm
- Note: Sufficient to test for $h < 0$

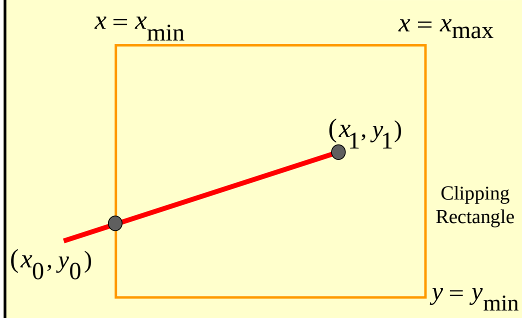
## End of Bresenham Circles



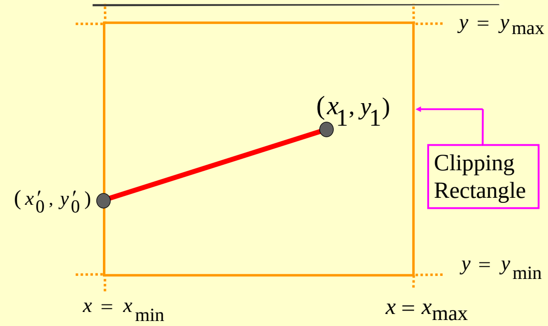
## Another Digital Line Issue

- Clipping Bresenham lines
- The integer slope is not the true slope
- Have to be careful
- More issues to follow

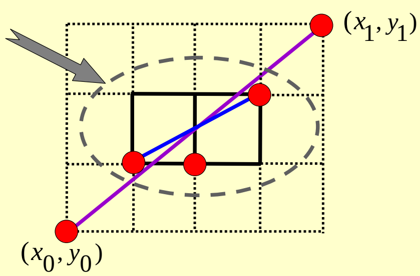
## Line Clipping Problem



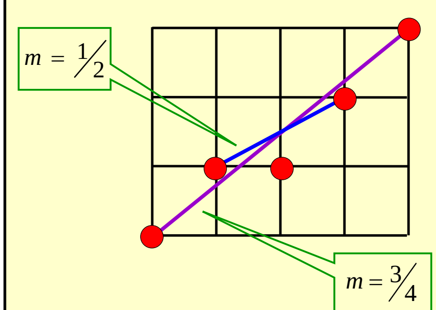

## Clipped Line



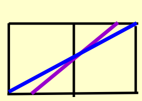

## Drawing Clipped Lines




## Clipped Line Has Different Slope !

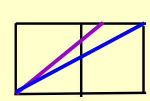



## Pick Right Slope to Reproduce Original Line Segment



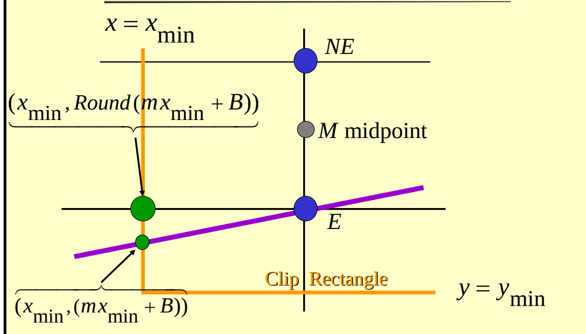
Zoom of previous situation

## Pick Right Slope to Reproduce Original Line Segment

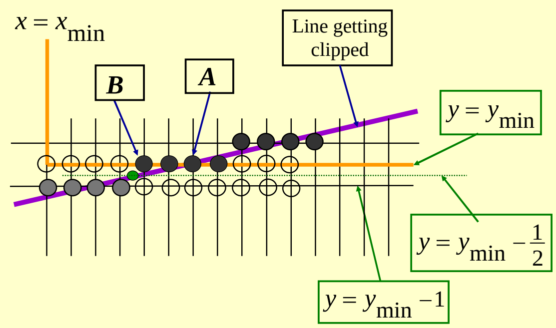


Zoom of previous situation

## Clipping Against $x = x_{min}$




## Clipping Against $y = y_{min}$



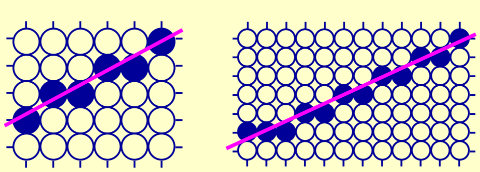

## Clipping Against $y = y_{min}$

- Situation is complicated
- Multiple pixels involved at $(y = y_{min})$
- Want all of those pixels as “*in*”
- Analytic $\cap$ , rounding $x$ gives $A$
- We want point $B$

## Clipping Against $y = y_{min}$

- Use $\boxed{Line} \cap \boxed{y = y_{min} - ½}$
- Round *up* to nearest integer $x$
- This yields point $B$, the desired result

## Jaggies-Manifestation of Aliasing



Added resolution helps, but does not directly address underlying issue of *aliasing*

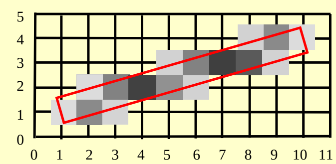
## Jaggies and Aliasing

- To represent a line with discrete pixel values is to sample finitely a continuous function
- Jaggies are visual manifestation, artifacts, resulting from information loss
- The term aliasing is a complicated, unintuitive phenomenon which will be defined later

## Jaggies and Aliasing

- Doubling resolution in x and y reduces the effect of the problem, but does not fix it
- Doubling resolution costs 4 times memory, memory bandwidth and scan conversion time!

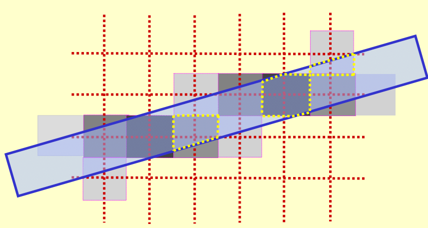
## Anti-aliasing



Pixel Space

Pixel intensity (darkness, in this case) is proportional to area covered by line

## Anti-aliasing



Pixel Space

Pixel intensity (darkness, in this case) is proportional to area covered by line

## Anti-aliasing

- Set each pixel's intensity value proportional to its area of overlap (i.e. sub-area) covered by primitive
- Not more than 1 pixel/column for lines with

$$0 < slope < 1$$

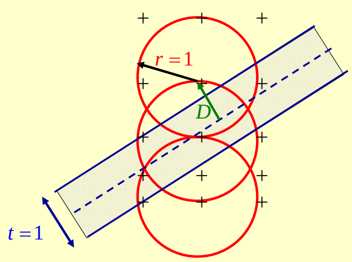
## *Gupta-Sproull Algorithm* -1

- Standard Bresenham chooses $E$ or $NE$
- Incrementally compute distance $D$ from chosen pixel to center of line
- Vary pixel intensity by value of $D$
- Do this for line above and below

## *Gupta-Sproull Algorithm* -2

- Use coarse (4-bit, say) lookup table for intensity : $Filter\ (D, t)$
- Note, $Filter$ value depends *only* on $D$ and $t$, not the slope of line! (Very clever)
- For *line_width* $t = 1$ geometry and associated calculations greatly simplify

## Cone Filter for Weighted Area Sampling




Unit thickness line intersects no more than 3 pixels

## Observations

- Lines are complicated
- Many aspects to consider
- We omitted many
- What about intensity of

  $y = x$ vs $y = 0$ ?

*The End*

Bresenham Circles

Lecture Set 3