Ray Tracing

Week 11

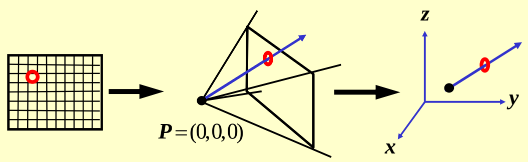
CS5600 ***Computer Graphics***

From Rich Riesenfeld
Spring 2013

Ray Tracing

- Classical geometric optics technique
- Extremely versatile
- Historically viewed as expensive
- Good for special effects
- Computationally intensive
- Can do sophisticated graphics

Ray Tracing



Screen-space point | Film plane point in canonical view volume | World-space ray

Environment Mapping

Ray Tracing Implementation - 1

- Key computation: Must find

  $ray \cap object$
- This is equivalent to

  $ray - object = 0$
- This is essentially *root finding*

Ray Tracing Implementation

- 2

- Ray is often represented parametrically,

  $\boldsymbol{r}(t) = t\,(\boldsymbol{P} - \boldsymbol{E})$,

  so we seek,

  $\boldsymbol{r}(t) \cap \boldsymbol{F}(x,y,z)$

- Problem requires intersection of parametric ray with some kind of surface
- Ray Tracing maps easily onto recursion

Ray Tracing Implementation

- 3

- RT’ing used for spectacular images
- RT’ing maps naturally to recursion
- RT’ing is trivially parallelized
- RT’ing has robustness problems
- RT’ing has aliasing problems

Ray Tracing

- 1

Three (*nonexclusive*) phenomena follow when ray intersects object:

1. *Reflect* (specularity)
2. *Pass through* (transparency)
3. *Stop* (diffuse - look for light vector and calculate proper value)

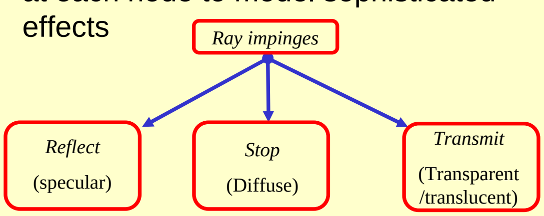
Ray Tracing

- 2

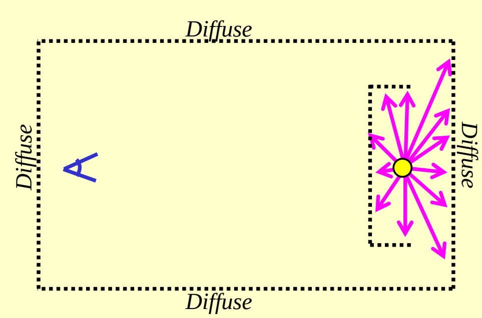
- Forward ray tracing: $E(S*)DL$
- Backward ray tracing: $L(S*)DE$
- What is the difference?

Ray Tracing (ternary) Tree

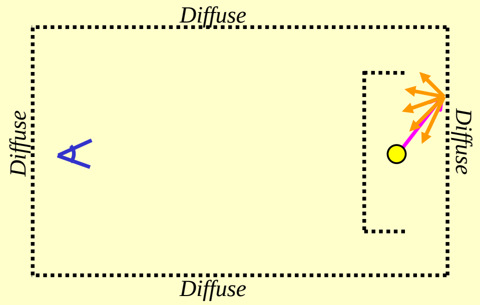
Often a combination of all three occur at each node to model sophisticated effects



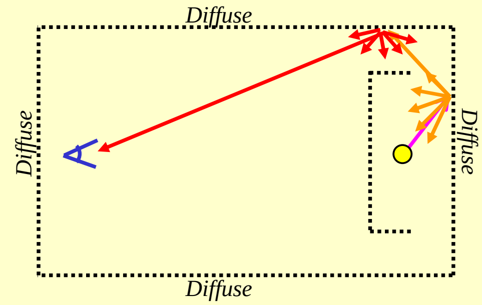
Ray Tracing (fwd)



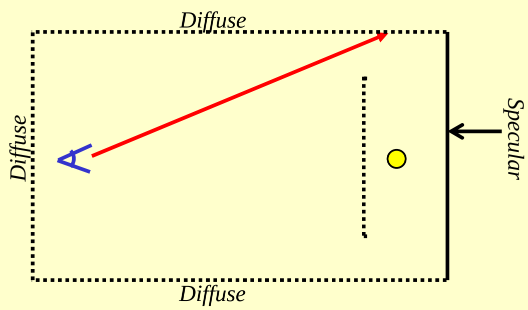
Ray Tracing (fwd)



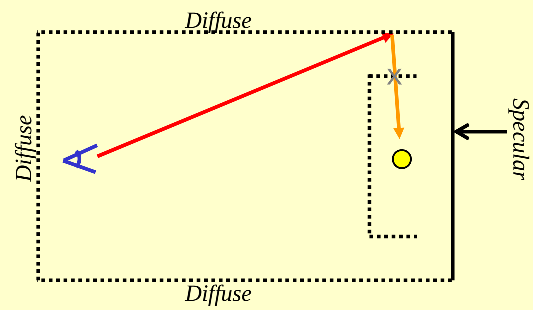
Ray Tracing (fwd)



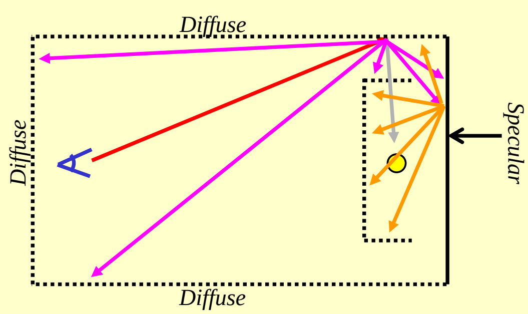
RT'ing Indirect Light (bkwd)



RT'ing Indirect Light (bkwd)



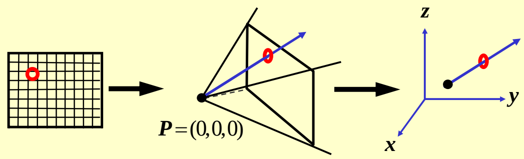
RT'ing Indirect Light (bkwd)



Ray Tracing Growth

- Tree can grow extremely fast, with high exponential fan out
- Fancy rays can have many cross-section geometries; not necessary a line with 0-cross-section
- Need to bound tree depth & fanout

Returning to Ray Tracing



Screen-space point — Film plane point in canonical view volume — World-space ray

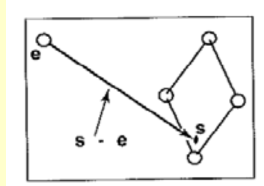
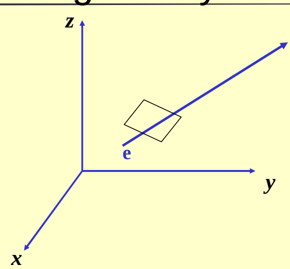


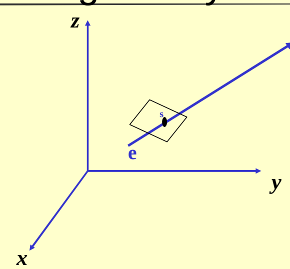
Figure 4.6. The ray from the eye to a point on the image plane.

Returning to Ray Tracing



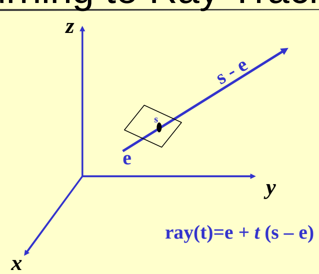
World-space ray

Returning to Ray Tracing



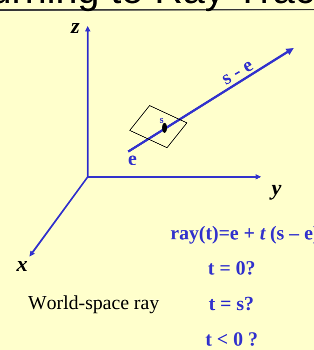
World-space ray

Returning to Ray Tracing



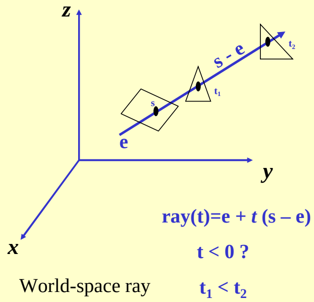
World-space ray

Returning to Ray Tracing

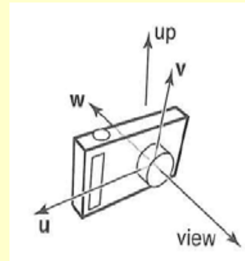


World-space ray

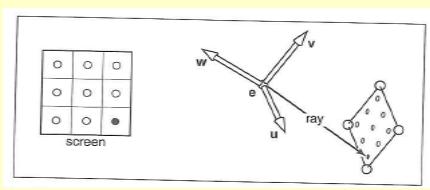
Returning to Ray Tracing



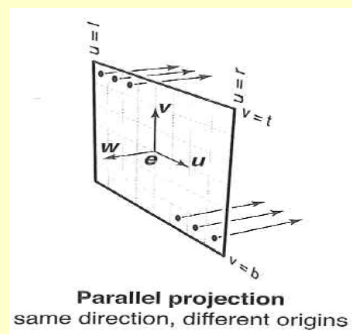
How to chose initial ray?



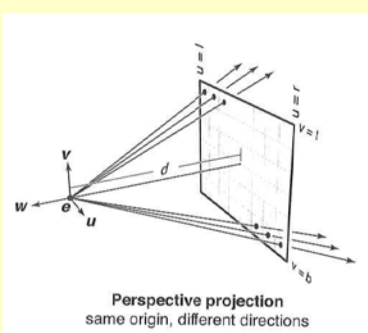
Rays through the pixels



Orthographic



Perspective



Returning to Ray Tracing

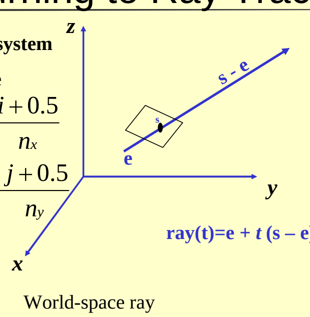
***uvw* coordinate system**

Origin is e

$$u_s = l + (r - l)\frac{i + 0.5}{n_x}$$

$$v_s = b + (t - b)\frac{j + 0.5}{n_y}$$

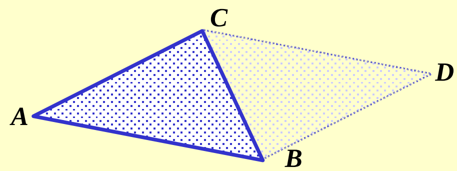
$$w_s = near$$



Computing Areas: Recall

$Area\ \square\ ABDC = AB \times AC$

$Area\ \square\ ABC = \frac{1}{2} AB \times AC$

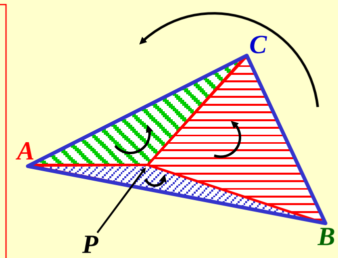


Barycentric Coords: Areas

$\alpha = \dfrac{Area\ \square\ PBC}{Area\ \square\ ABC}$

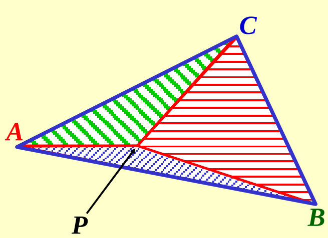
$\beta = \dfrac{Area\ \square\ PCA}{Area\ \square\ ABC}$

$\gamma = \dfrac{Area\ \square\ PAB}{Area\ \square\ ABC}$



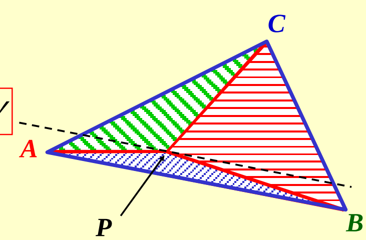
Barycentric Coords: Areas

Then,
$\alpha, \beta, \gamma \geq 0,$
$\alpha + \beta + \gamma \equiv 1,$
and, $P =$
$\alpha A + \beta B + \gamma C$



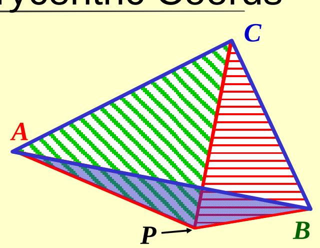
Isoparametric lines

constant γ



Extended Barycentric Coords

Then,
~~$\alpha, \beta, \gamma \geq 0,$~~
$\alpha + \beta + \gamma \equiv 1,$
and, $\boldsymbol{P} =$
$\alpha \boldsymbol{A} + \beta \boldsymbol{B} + \gamma \boldsymbol{C}$

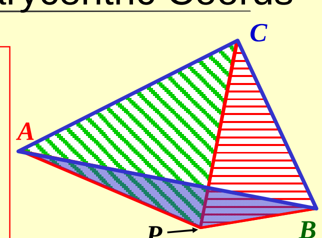


P can be *outside* triangle if coord's allowed to go negative.

Extended Barycentric Coords

Can use
$\alpha, \beta, \gamma \leq 0$?

for test of $\boldsymbol{P}$
outside $\square$ ***ABC*** ?

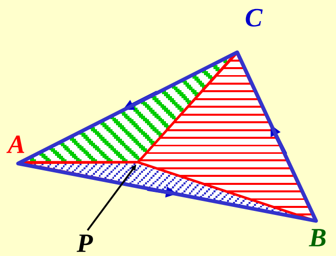


P can be *outside* triangle, but in it plane, if coord's can be negative.

Algorithm for Computing Barycentric Coords of P

- 1

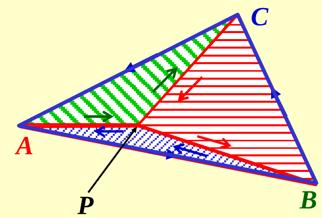
1. As indicated, assign an orientation to □ABC



Algorithm for Computing Barycentric Coords of P

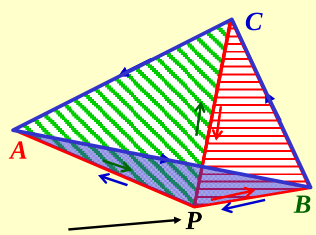
- 2

2. Compute all areas using □PAB,□PBC and □PCA Orientation is important!



Algorithm for Computing Barycentric Coords of P

- 3



Note this works equally for P *outside*

Algorithm for Computing Barycentric Coords of P

- 4

3. Compute all areas using cross products in the following manner,

$$2*Area_{big} = \|\mathbf{AB}\times\mathbf{BC}\|$$
$$2*Area_{\alpha} = \|\mathbf{PB}\times\mathbf{BC}\|$$
$$2*Area_{\beta} = \|\mathbf{PC}\times\mathbf{CA}\|$$
$$2*Area_{\gamma} = \|\mathbf{PA}\times\mathbf{AB}\|$$

Algorithm for Computing Barycentric Coords of P

- 5

4. Now compute ratios for α,β,γ

$$\alpha = \frac{2*Area_{\alpha}}{2*Area_{big}} = \frac{\|\mathbf{PB}\times\mathbf{BC}\|}{\|\mathbf{AB}\times\mathbf{BC}\|}$$
$$\beta = \frac{2*Area_{\alpha}}{2*Area_{big}} = \frac{\|\mathbf{PC}\times\mathbf{CA}\|}{\|\mathbf{AB}\times\mathbf{BC}\|}$$
$$\gamma = \frac{2*Area_{\alpha}}{2*Area_{big}} = \frac{\|\mathbf{PA}\times\mathbf{AC}\|}{\|\mathbf{AB}\times\mathbf{BC}\|}$$

Algorithm for Computing Barycentric Coords of P

- 6

5. Now compute sign of area. *In* or *out*? Look at *dot product* of area vectors! P is *in* if same *sign*, i.e., positive; *out*, outwise.

$$Sign_{\alpha} = sign\{(\mathbf{PB}\times\mathbf{BC})\cdot(\mathbf{AB}\times\mathbf{BC})\}$$
$$Sign_{\beta} = sign\{(\mathbf{PC}\times\mathbf{CB})\cdot(\mathbf{AB}\times\mathbf{BC})\}$$
$$Sign_{\gamma} = sign\{(\mathbf{PA}\times\mathbf{AC})\cdot(\mathbf{AB}\times\mathbf{BC})\}$$

Applet

Universität Karlsruhe (TH)
Geometrische Datenverarbeitung:

<http://i33www.ira.uka.de/applets/mocca/html/noplugin/inhalt.html>

Recall Cramer's Rule - 1

Let, $\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

That is, $\boldsymbol{Mx} = \boldsymbol{y}$

and, $\|\boldsymbol{M}\| = \begin{Vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{Vmatrix} = \det(\boldsymbol{M})$

Recall Cramer's Rule - 2

Then,

$x_1 = \frac{1}{\|\boldsymbol{M}\|}\begin{Vmatrix} y_1 & m_{12} & m_{13} \\ y_2 & m_{22} & m_{23} \\ y_3 & m_{32} & m_{33} \end{Vmatrix}; \quad x_2 = \frac{1}{\|\boldsymbol{M}\|}\begin{Vmatrix} m_{11} & y_1 & m_{13} \\ m_{21} & y_2 & m_{23} \\ m_{31} & y_3 & m_{33} \end{Vmatrix};$

and,

$x_3 = \frac{1}{\|\boldsymbol{M}\|}\begin{Vmatrix} m_{11} & m_{12} & y_1 \\ m_{21} & m_{22} & y_2 \\ m_{31} & m_{32} & y_3 \end{Vmatrix}$

Ray Intersect Parametric Object

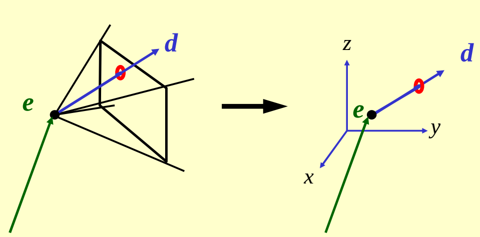
Let $\boldsymbol{S}(u,v) = \left(f(u,v), g(u,v), h(u,v)\right)$

be a parametrically defined object.

Componentwise, this means,

$$e_x + td_x = f(u,v)$$
$$e_y + td_y = g(u,v)$$
$$e_z + td_z = h(u,v)$$

Ray Tracing



Barycentric Coords: Areas

Then,
$\alpha, \beta, \gamma \geq 0,$
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and, $\boldsymbol{P} =$
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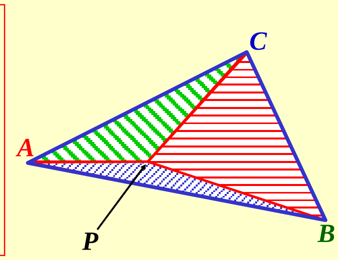
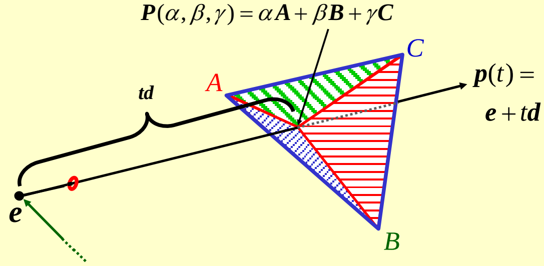


Solving for $\boldsymbol{p}(t) \cap \triangle \boldsymbol{ABC}$

$\boldsymbol{P}(\alpha,\beta,\gamma) = \alpha\boldsymbol{A} + \beta\boldsymbol{B} + \gamma\boldsymbol{C}$



Ray Intersect Parametric Object

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$$e_x + td_x = f(u,v)$$
$$e_y + td_y = g(u,v)$$
$$e_z + td_z = h(u,v)$$

Shirley's $ray \cap \triangle$ Method - 1

We can think of this form as establishing the origin at $\boldsymbol{A}$, and using basis vectors $\{(\boldsymbol{B}-\boldsymbol{A}), (\boldsymbol{C}-\boldsymbol{A})\}$ to span the plane of $\triangle \boldsymbol{ABC}$:

$$\boldsymbol{P}(\beta,\gamma) = \boldsymbol{A} + \beta(\boldsymbol{B}-\boldsymbol{A}) + \gamma(\boldsymbol{C}-\boldsymbol{A})$$

Shirley's $ray \cap \triangle$ Method - 2

Let $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$ be a ray, and define the plane as

$$\boldsymbol{P}(\alpha,\beta,\gamma) = \alpha\boldsymbol{A} + \beta\boldsymbol{B} + \gamma\boldsymbol{C},$$

or, $\boldsymbol{P}(\beta,\gamma) = \overbrace{(1-\beta-\gamma)}\boldsymbol{A} + \beta\boldsymbol{B} + \gamma\boldsymbol{C}$

$$= \boldsymbol{A} + \beta(\boldsymbol{B}-\boldsymbol{A}) + \gamma(\boldsymbol{C}-\boldsymbol{A})$$

Shirley's $ray \cap \triangle$ Method - 3

Where do ray $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$, and plane defined as

$$\boldsymbol{P}(\alpha,\beta,\gamma) = \alpha\boldsymbol{A} + \beta\boldsymbol{B} + \gamma\boldsymbol{C},$$

intersect?

$$\boldsymbol{A} + \beta(\boldsymbol{B}-\boldsymbol{A}) + \gamma(\boldsymbol{C}-\boldsymbol{A}) = \boldsymbol{e} + t\boldsymbol{d}$$
$$\boldsymbol{A} - \boldsymbol{e} = -\beta(\boldsymbol{B}-\boldsymbol{A}) - \gamma(\boldsymbol{C}-\boldsymbol{A}) + t\boldsymbol{d}$$
$$\beta(\boldsymbol{A}-\boldsymbol{B}) + \gamma(\boldsymbol{A}-\boldsymbol{C}) + t\boldsymbol{d} = \boldsymbol{A} - \boldsymbol{e}$$

Shirley's $ray \cap \triangle$ Method - 4

In matrix form

$$\beta(\boldsymbol{A}-\boldsymbol{B}) + \gamma(\boldsymbol{A}-\boldsymbol{C}) + t\boldsymbol{d} = \boldsymbol{A} - \boldsymbol{e}$$

$$\left[(\boldsymbol{A}-\boldsymbol{B}) \quad (\boldsymbol{A}-\boldsymbol{C}) \quad \boldsymbol{d}\right] \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \boldsymbol{A} - \boldsymbol{e}$$

Ray Triangle

$$e + td = a + \beta(b-a) + \gamma(c-a)$$

$$x_e + tx_d = x_a + \beta(x_b - x_a) + \gamma(x_c - x_a)$$
$$y_e + ty_d = y_a + \beta(y_b - y_a) + \gamma(y_c - y_a)$$
$$z_e + tz_d = z_a + \beta(z_b - z_a) + \gamma(z_c - z_a)$$

Ray Triangle

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

Shirley's *ray* ∩ □ Method [- 5]

And,

$$\begin{bmatrix} (\mathbf{A}-\mathbf{B}) & (\mathbf{A}-\mathbf{C}) & \mathbf{d} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \mathbf{A} - \mathbf{e}$$

$$\overbrace{\begin{bmatrix} (x_a - x_b) & (x_a - x_c) & x_d \\ (y_a - y_b) & (y_a - y_c) & y_d \\ (z_a - z_b) & (z_a - z_c) & z_d \end{bmatrix}}^{\boldsymbol{M}} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

By Cramer's Rule [- 1]

$$\beta = \frac{1}{\|M\|} \begin{Vmatrix} (x_a - x_e) & (x_a - x_c) & x_d \\ (y_a - y_e) & (y_a - y_c) & y_d \\ (z_a - z_e) & (z_a - z_c) & z_d \end{Vmatrix}$$

$$\gamma = \frac{1}{\|M\|} \begin{Vmatrix} (x_a - x_b) & (x_a - x_e) & x_d \\ (y_a - y_b) & (y_a - y_e) & y_d \\ (z_a - z_b) & (z_a - z_e) & z_d \end{Vmatrix}$$

By Cramer's Rule [- 2]

$$t = \frac{1}{\|M\|} \begin{Vmatrix} (x_a - x_b) & (x_a - x_c) & (x_a - x_e) \\ (y_a - y_b) & (y_a - y_c) & (y_a - y_e) \\ (z_a - z_b) & (z_a - z_c) & (z_a - z_e) \end{Vmatrix}$$

Shirley's *ray* ∩ □ Method

Let $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$ be a ray.

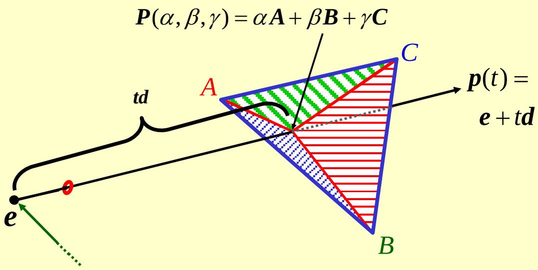
For any object defined by f,

we are looking for

$$\mathrm{f}\left(\boldsymbol{p}(t)\right) = \mathrm{f}\left(\boldsymbol{e} + t\boldsymbol{d}\right) = 0,$$

for intersection points

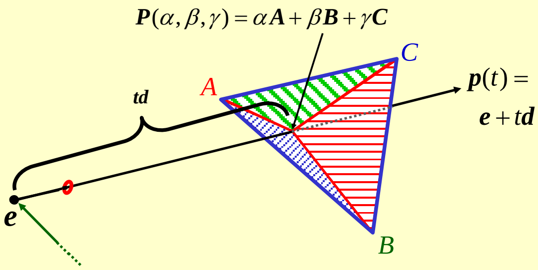
Solving for $p(t) \cap \triangle ABC$



What about texturing?

We know the intersection point, *p*, what about (*s*,*t*)?

Solving for $p(t) \cap \triangle ABC$



$ray \cap implicit\_object$

Let $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$ be a ray.

For any object defined by

$f(x, y, z) = 0$, we are looking for

$f\left(\boldsymbol{p}(t)\right) = f\left(\boldsymbol{e} + t\boldsymbol{d}\right) = 0,$

for intersection points

Shirley's $ray \cap$ Polygon Method

Let $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$ be a ray.

For any object defined by f,

we are looking for

$\mathrm{f}\left(\boldsymbol{p}(t)\right) = \mathrm{f}\left(\boldsymbol{e} + t\boldsymbol{d}\right) = 0,$

for intersection points

Shirley's $ray \cap$ Polygon Method

*f (p(t)) = f (e + t**d**) = 0*

Equation to test point in plane:

$(p - p_1) \text{ dot } n = 0$

Plug ray-equ into plane-equ:

$(e + t\mathbf{d} - p_1) \text{ dot } n = 0$

Shirley's $ray \cap$ Polygon Method

(e + t**d** – p_1) dot n = 0
(t**d** + e – p_1) dot n = 0
(t**d** dot n) + (e – p_1) dot n = 0
t**d** dot n = (p_1 – e) dot n
t = (p_1 – e) dot n / **d** dot n
Need to check if p in inside polygon

$ray \cap implicit\_object$

Let $\boldsymbol{p}(t) = \boldsymbol{e} + t\boldsymbol{d}$ be a ray.
For any object defined by
$f(x, y, z) = 0$, we are looking for
$f\left(\boldsymbol{p}(t)\right) = f\left(\boldsymbol{e} + t\boldsymbol{d}\right) = 0$,
for intersection points

$ray \cap sphere$

- 1

For sphere with center $\boldsymbol{c} = (c_x, c_y, c_z)$,

$$(\boldsymbol{p} - \boldsymbol{c}) \bullet (\boldsymbol{p} - \boldsymbol{c}) - R^2 = 0$$

is a vector eq with $\boldsymbol{p} = (p_x, p_y, p_z)$.

$$\therefore \quad (\boldsymbol{e} + t\boldsymbol{d} - \boldsymbol{c}) \bullet (\boldsymbol{e} + t\boldsymbol{d} - \boldsymbol{c}) - R^2 = 0$$

$ray \cap sphere$

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Collecting terms,

$$\boldsymbol{d} \cdot \boldsymbol{d}\, t^2 + 2\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c})t + (\boldsymbol{e} - \boldsymbol{c}) \cdot (\boldsymbol{e} - \boldsymbol{c}) - R^2 = 0$$

Hence, $t =$

$$\frac{-\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c}) \pm \sqrt{(\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c}))^2 - (\boldsymbol{d} \cdot \boldsymbol{d})\left((\boldsymbol{e} - \boldsymbol{c}) \cdot (\boldsymbol{e} - \boldsymbol{c}) - R^2\right)}}{(\boldsymbol{d} \cdot \boldsymbol{d})}$$

And, unit normal is $\boldsymbol{n} = (\boldsymbol{p} - \boldsymbol{c}) / R$

$ray \cap sphere$

Sign of: $(\boldsymbol{d} \cdot (\boldsymbol{e} - \boldsymbol{c}))^2 - (\boldsymbol{d} \cdot \boldsymbol{d})\left((\boldsymbol{e} - \boldsymbol{c}) \cdot (\boldsymbol{e} - \boldsymbol{c}) - R^2\right)$

if sign < 0 : ray misses sphere
sign = 0: ray is tangent to sphere
sign > 0: 2 intersections

Which to use?

Lecture Week 12 B

End

Ray Tracing B

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