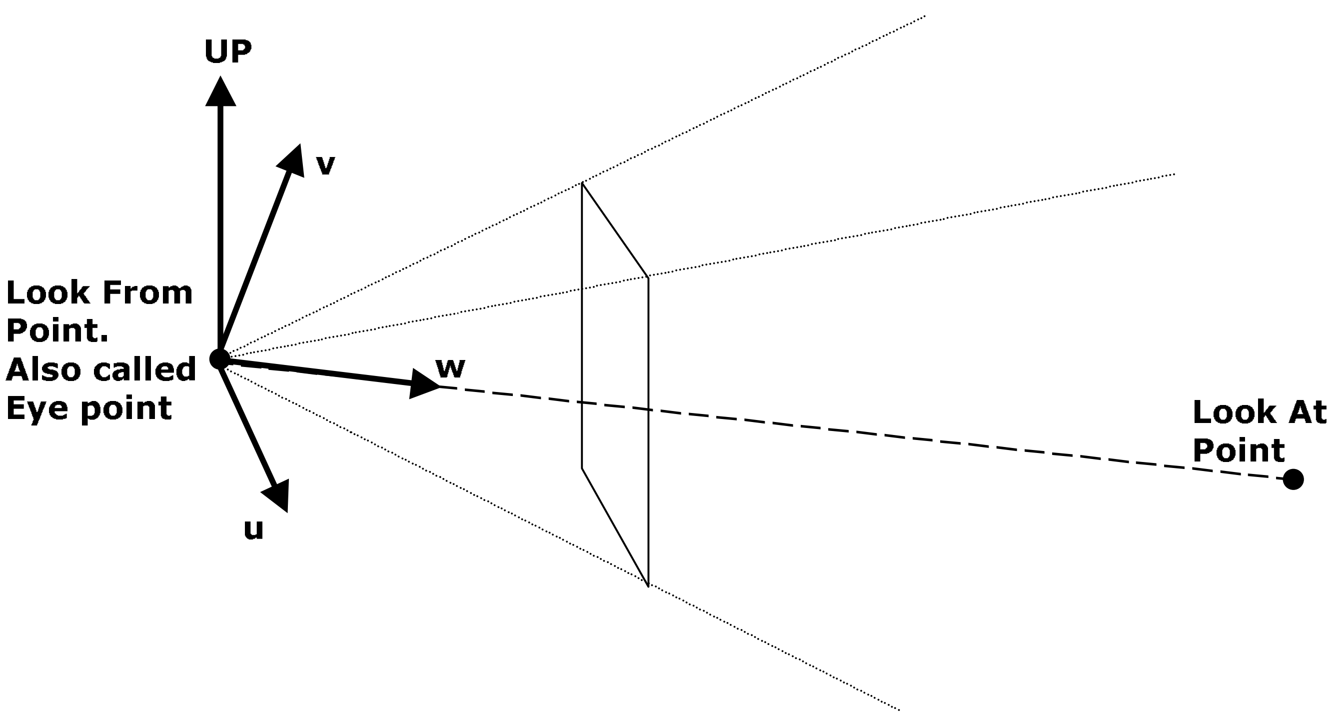
Perspective Transformations

Viewing system matrix $\mathbf{M}_{sys}$ transform is obtained by combining the view matrix with the perspective projection with the viewport to screen matrix. These are defined as:

$$\mathbf{M}_{sys} = \mathbf{M}_{screen}\ \mathbf{M}_{perspective}\ \mathbf{M}_{view}$$

View Matrix

We want to compute the view matrix that aligns the orthonormal basis at the origin and pointing down either the +Z (right-handed) or –Z (left-handed). Here's the picture:



To form the view transform, the axes for the coordinate system of are given by (for right hand system):

$$W = \frac{eye - at}{\| eye - at \|}$$

$$U = \frac{up \times w}{\| up \times w \|}$$

$$V = \frac{w \times u}{\| w \times u \|}$$

For a left-handed system:

$$W = \frac{at - eye}{\| at - eye \|}$$

$$U = \frac{up \times w}{\| up \times w \|}$$

$$V = \frac{u \times w}{\| u \times w \|}$$

$$\begin{vmatrix} U_x & U_y & U_z & 0 \\ V_x & V_y & V_z & 0 \\ W_x & W_y & W_z & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Orthonormal Rotation about origin

$$\begin{vmatrix} 1 & 0 & 0 & -eye_x \\ 0 & 1 & 0 & -eye_y \\ 0 & 0 & 1 & -eye_z \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Translation to origin

$$\begin{vmatrix} U_x & U_y & U_z & 0 \\ V_x & V_y & V_z & 0 \\ W_x & W_y & W_z & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} 1 & 0 & 0 & -eye_x \\ 0 & 1 & 0 & -eye_y \\ 0 & 0 & 1 & -eye_z \\ 0 & 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} VERTEX_x \\ VERTEX_y \\ VERTEX_z \\ 1 \end{vmatrix}$$