Suppose that

```
{with \{x \ 88\} \ \{+ \ x \ y\}\}
```

appears in a program; the body is eventually evaluated:

where will x be in the substitution?

**Answer:** always at the beginning:

Suppose that

$$\{with \{y 1\} \{+ x y\}\}$$

appears in a program; the body is eventually evaluated:

where will y be in the substitution?

**Answer:** always at the beginning:

Suppose that

```
{with {y 1}
  {with {x 2} {+ x y}}}
```

appears in a program; the body is eventually evaluated:

where will y be in the substitution?

**Answer:** always second:

$$x = 2$$
  $y = 1$  ...

### Suppose that

```
{with {y 1}
  {with {x 88} {- {+ x y} 17}}}
```

appears in a program; the body is eventually evaluated:

where will x and y be in the substitution?

**Answer:** always first and second:

$$x = 88 \qquad y = 1 \qquad \dots$$

Suppose that

appears in a program; the body is eventually evaluated:

where will x and y be in the substitution?

**Answer:** always first and fourth:

```
x = 0 z = 9 w = 10 y = 1 ...
```

#### Suppose that

appears in a program; the body is eventually evaluated:

where will x and y be in the substitution?

**Answer:** always first and fourth:

$$x = 0$$
  $z = 9$   $w = 10$   $y = 1$  ...

## Lexical Scope

#### Our language is **lexically scoped**:

- For any expression, we can tell which identifiers will have substititions at run time
- The order of the substitutions is also predictable

# Compiling FIWAE

A **compiler** can transform an **FW1AE** expression to an expression without identifiers — only lexical addresses

```
; compile : F1WAE ... -> CF1WAE
```

```
(define-type F1WAE
                               (define-type CF1WAE
  [num (n number?)]
                                  [cnum (n number?)]
                                  [cadd (lhs CF1WAE?)
  [add (lhs F1WAE?)
      (rhs F1WAE?)]
                                        (rhs CF1WAE?)]
  [sub (lhs F1WAE?)
                                  [csub (lhs CF1WAE?)
       (rhs F1WAE?)]
                                        (rhs CF1WAE?)]
  [with (name symbol?)
                                  [cwith (named-expr CF1WAE?)
                                         (body CF1WAE?)]
        (named-expr F1WAE?)
        (body F1WAE?)]
                                  [cat (pos number?)]
  [id (name symbol?)]
                                  [capp (fun-name symbol?)
  [app (fun-name symbol?)
                                        (arg-expr CF1WAE?)])
       (arg-expr F1WAE?)])
```

### Compile Examples

```
(compile |1| \ldots) \Rightarrow |1|
 (compile | \{+12\} | \ldots \rangle \Rightarrow | \{+12\} |
 (compile |x| \dots) \Rightarrow compile: free identifier
(compile | {with {x 8} x} | ...) \Rightarrow {with 8 {at 0}}
(compile | {with {y 1} {with {x 2} {+ x y}}} | ...)
  ⇒ {with 1 {with 2 {+ {at 0} {at 1}}}}
 (compile | {deffun {f x} x} | ...)
   ⇒ {deffun f {at 0}}
```

### Implementing the Compiler

```
; compile : F1WAE CSub -> CF1WAE
(define (compile a-wae cs)
  (type-case F1WAE a-wae
    [num (n) (cnum n)]
    [add (1 r) (cadd (compile 1 cs)
                      (compile r cs))]
    [sub (1 r) (csub (compile 1 cs)
                      (compile r cs))]
    [with (named named-expr body-expr)
      (cwith (compile named-expr cs)
             (compile body-expr
                       (aCSub named cs)))]
    [id (name) (cat (locate name cs))]
    [app (fun-name arg-expr)
         (capp fun-name
               (compile arg-expr cs))]))
```

### Compile-Time Substitution

Mimics run-time substitutions, but without values:

```
(define-type CSub
  [mtCSub]
  [aCSub (name symbol?)
         (rest CSub?)])
; locate : symbol CSub -> number
(define (locate name cs)
  (type-case CSub cs
    [mtCSub ()
            (error 'compile "free identifier")]
    [aCSub (sub-name rest)
           (if (symbol=? name sub-name)
               (+ 1 (locate name rest)))]))
```

## **CFIWAE** Interpreter

Almost the same as **F1WAE** interp:

```
; cinterp : CF1WAE list-of-num -> num
(define (cinterp a-cwae s)
  (type-case CF1WAE a-cwae
    [cnum (n) n]
    [cadd (l r) (+ (cinterp l s) (cinterp r s))]
    [csub (1 r) (- (cinterp 1 s) (cinterp r s))]
    [cwith (named-expr body-expr)
           (cinterp body-expr
                    cfundefs
                    (cons (cinterp named-expr cfundefs s)
                          s))1
    [cat (pos) (list-ref s pos)]
    [capp (fun-name arg)
          (local [(define fun (lookup-cfundef fun-name cfundefs))
                  (define arg-val (cinterp arg cfundefs s))]
            (cinterp (cfundef-body fun)
                     cfundefs
                     (cons arg-val empty)))))
```

## CFIWAE Versus FIWAE Interpretation

#### On my machine,

```
(cinterp
{with {x 1} {with {y 2} {with {z 3} {+ {+ x x} {+ x x}}}}}
empty)
```

#### takes about half the time of

Note: using built-in **list-ref** simulates machine array indexing, but don't take the numbers too seriously