

Using the same type rules as in the practice midterm's question 2, show a derivation tree for each of the following expressions.

1. $\{\text{lambda } \{[x : \text{num}] \} x\}$

2. $\{\text{lambda } \{[x : \text{num}] \} \{+ x 1\}\}$

3. $\{\text{lambda } \{[x : (\text{num} \rightarrow \text{num})] \} \{x 1\}\}$

4. $\{\{\text{lambda } \{[x : (\text{num} \rightarrow \text{num})] \} \{x 1\}\} \{\text{lambda } \{[z : \text{num}] \} \{+ z 2\}\}\}$

Answers

1.

$$\frac{[x \leftarrow \text{num}] \vdash x : \text{num}}{\emptyset \vdash \{\text{lambda } \{[x : \text{num}]\} x\} : (\text{num} \rightarrow \text{num})}$$

2.

$$\frac{\frac{\Gamma_1 \vdash x : \text{num} \quad \Gamma_1 \vdash 1 : \text{num}}{\Gamma_1 = [x \leftarrow \text{num}] \vdash \{+ x 1\} : \text{num}}}{\emptyset \vdash \{\text{lambda } \{[x : \text{num}]\} \{+ x 1\}\} : (\text{num} \rightarrow \text{num})}$$

3.

$$\frac{\frac{\Gamma_1 \vdash x : (\text{num} \rightarrow \text{num}) \quad \Gamma_1 \vdash 1 : \text{num}}{\Gamma_1 = [x \leftarrow (\text{num} \rightarrow \text{num})] \vdash \{x 1\} : \text{num}}}{\emptyset \vdash \{\text{lambda } \{[x : (\text{num} \rightarrow \text{num})]\} \{x 1\}\} : ((\text{num} \rightarrow \text{num}) \rightarrow \text{num})}$$

4.

$$\frac{\frac{\frac{\Gamma_1 \vdash x : (\text{num} \rightarrow \text{num}) \quad \Gamma_1 \vdash 1 : \text{num}}{\Gamma_1 = [x \leftarrow (\text{num} \rightarrow \text{num})] \vdash \{x 1\} : \text{num}}}{\emptyset \vdash \{\lambda \{[x : (\text{num} \rightarrow \text{num})]\} \{x 1\}\} : ((\text{num} \rightarrow \text{num}) \rightarrow \text{num})} \quad \frac{\frac{\Gamma_2 \vdash z : \text{num} \quad \Gamma_2 \vdash 2 : \text{num}}{\Gamma_2 = [z \leftarrow \text{num}] \vdash \{+ z 2\} : \text{num}}}{\emptyset \vdash \{\lambda \{[z : \text{num}]\} \{+ z 2\}\} : (\text{num} \rightarrow \text{num})}}{\emptyset \vdash \{\{\lambda \{[x : (\text{num} \rightarrow \text{num})]\} \{x 1\}\} \lambda \{[z : \text{num}]\} \{+ z 2\}\} : \text{num}}$$