Name:

Practice Final Exam

Instructions: You may use any notes that you like, but no calculators or computers are allowed. Be sure to show all of your work. It is not necessary to simplify your answers.

Note: You may also want to see the practice exams for the ECE class:

http://www.ece.utah.edu/~ece3530/

1. It is estimated that 6 out of every 1,000 people have autism spectrum disorder, i.e., there is a 0.6% chance of being born with the disorder. Of people with autism spectrum disorder, 80% are male. Let A be the event that a person has autism spectrum disorder, and M be the event that a person is a male. Also, assume in this problem that it is equally likely to be born male or female, that is, $P(M) = P(M^c) = 0.5$.

   (a) What does $P(M \mid A)$ mean in English? What is its value?

   (b) What does $P(A \mid M)$ mean in English? What is its value?

   (c) What is the probability of being a female with autism spectrum disorder? (First write down the probability expression in terms of A and M, and then compute.)

2. Let X be a geometric random variable, with probability p of success. Hint: you will need the following identity **somewhere** in this problem:

$$\sum_{i=1}^{n}(1-p)^{i-1}p = 1 - (1-p)^n.$$

(a) What is the probability that the first success is not on the first attempt? In other words, what is $P(X > 1)$?

(b) What is $P(X \leq n)$?

(c) What is $P(X > n)$, i.e., the probability of starting on a losing streak of length n? Hint: use your answer in (b). Also, check that this matches (a) when $n = 1$.

(d) Show that starting on a losing streak does not change the probability of continuing a losing streak. That is, use your result in (c) to show that

$$P(X = k + n \,|\, X > n) = P(X = k).$$

3. Consider a random variable X with cummulative distribution function (cdf):

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ k\,(x + \sin(x)) & \text{for } 0 \le x \le \pi \\ 1 & \text{for } x > \pi \end{cases}$$

(a) What is the value for k that makes F_X a valid cdf?

(b) What is the probability density function (pdf) for X?

(c) What is $P\left(X \le \frac{\pi}{2}\right)$

(d) What is the integral to compute $E[X]$? (you don't need to solve it)

4. Say you are given two random variables X and Y that have the same variance, $\text{Var}(X) = \text{Var}(Y) = \sigma^2$.

(a) If X and Y are uncorrelated, what is $\text{Var}(X + Y)$?

(b) If X and Y are negatively correlated with $\rho(X, Y) = -1$, what is $\text{Var}(X + Y)$?

(c) For both parts (a) and (b) above, state whether X and Y are dependent, independent, or "not enough information to tell". Explain your answers.

5. A baseball player has a batting average of 0.305 after 100 at-bats. Use a normal approximation to give a 99% confidence interval for the player's true probability of hitting safely. To be conservative, you should use a "worst case" value of $\sigma = 0.25$ in your formula (this is the largest that σ can get for an estimate of a percentage). **You just need to set up the formula and plug the correct numbers into it, you don't need to do the arithmetic.** You will want to use **one** of the following values:

$$z_{0.05} = 1.64, \qquad z_{0.025} = 1.96, \qquad z_{0.005} = 2.58, \qquad z_{0.01} = 2.33$$

6. You've started a company that creates phone apps. At your current price point, you need to sell 800 apps per month to stay profitable. Over the last 4 months, you have sold 805 apps on average each month. The sample standard deviation during that time period is 10 apps.

   (a) Set up a hypothesis test that your average monthly sales exceed the goal. What is your null hypothesis and alternate hypothesis?

   (b) Let F denote the cdf for the appropriate t distribution. Using a level of $\alpha = 0.05$, circle one of the options below for your critical value:

   $$F(0.05) = 0.518, \quad F^{-1}(0.05) = -2.53, \quad F(0.95) = 0.794, \quad F^{-1}(0.95) = 2.35$$

   (c) Compute the statistic you would use to test the hypothesis.
   Hint: its a simple number!

   (d) Would you reject the null hypothesis at the $\alpha = 0.05$ level?

   Circle one: Yes   or   No