

Homework 1: Getting Started With Probability

Instructions: Submit a single R Markdown file (.Rmd) of your work on Canvas by 11:59pm on the due date. You may also submit diagrams, drawings, etc. as image files (.png, .jpg, .gif)—they must be formatted into your .Rmd document (we won't look at them separately). **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Do not look at another student's answers, and do not show your answers to anyone.

1. Say 16% of the population is rich, 10% of the population is famous, and 8% of the population is both rich and famous. Define events R = "person is rich" and F = "person is famous" for some randomly selected person in the population. Write an expression for each of the following events using set operations involving the events R and F . **Here you can just give the answer, and do not need to show any work.**

Example: "The person is rich and famous" would be the event $R \cap F$.

- (a) The person is not famous.
 - (b) The person is rich but not famous.
 - (c) The person is either rich or famous (or both).
 - (d) The person is neither rich nor famous.
2. **(16 pts)** Calculate the probabilities for each of the four events in the previous problem. **Be sure to show your intermediate steps and list any probability rules that you use.**
Example: For the "rich and famous" event, the probability is $P(R \cap F) = 0.08$
 3. Say there are two events A and B , with $B \subset A$, and you know $P(A)$ and $P(B)$.
 - (a) What is a simple expression for $P(A \cap B^c)$, using no more information than just $P(A)$ and $P(B)$? **Show your work by listing intermediate equations or by drawing a Venn diagram.**
 - (b) Now say you are testing a piece of software, and define events A = "the software crashes" and B = "the software crashes because of a segmentation fault". In English, what is the event $A \cap B^c$?
 - (c) If $P(A) = 0.05$ (probability of crash) and $P(B) = 0.02$ (probability of segmentation fault), then what is $P(A \cap B^c)$? Use your formula from part (a).
 4. Exercise 2.6 from the book. **Be sure to name any probability rules that you use.**
 5. You have two \$1 bills, two \$5 bills, and two \$10 bills. If you pick two bills at random (without looking!), what is the probability that they are the same denomination? **Explain how you arrived at the answer, don't just give a number!**

6. You have a deck of 16 cards, with:

- 6 cards that are white on both sides;
- 4 cards that are white on one side, black on the other;
- 6 cards that are black on both sides.

The cards are shuffled and randomly flipped. You draw one card at random from the deck and look only at one side of it. *Hint:* Be careful! Each side of the card counts as a separate outcome. Think about it in terms of the 32 sides that are possible (not the 16 possible cards).

- (a) Draw a tree diagram with the probabilities of all possible card types. **Your tree should have two levels, one for the color of the top of the card, and one for the color of the bottom of the card. It should also include all of the joint probabilities (top color AND bottom color) at the ends.**
- (b) What is the probability that the top of your card is white?
- (c) If the top of your card is white, what is the probability that the bottom is black?
- (d) What is the probability that the bottom of your card is black? **Show how you got this.**