ELECTROMAGNETICS
MF ISKANDER ©

4.8.3 Capacitance Calculation Using Finite Difference

The question that is often asked by students is how to relate the calculated values of the node potential to quantities of engineering interest, such as the capacitance of a system of conductors of complex geometry. Calculation of the capacitance from the obtained potential distribution may be done through Gauss's law, as follows:

$$\oint_s \epsilon \mathbf{E} \cdot d\mathbf{s} = q = -\oint_s \epsilon \nabla\Phi \cdot d\mathbf{s} \tag{4.56}$$

In equation 4.56, the electric field **E** was substituted by the electric potential $\mathbf{E} = -\nabla\Phi$. In a two-dimensional problem such as the conductor arrangement shown in Figure 4.22, the potential is independent of the axial coordinate, and the closed surface s may be replaced by the closed contour c. Hence,

$$-\oint_s \epsilon \nabla\Phi \cdot d\mathbf{c} = -\oint_c \epsilon \frac{\partial\Phi}{\partial n} dc = q \tag{4.57}$$

where q in this case is the charge per unit length in coulombs per meter. $\nabla\Phi \cdot d\mathbf{c}$ in equation 4.57 is replaced by the normal derivative $\partial\Phi/\partial n$ integrated over the closed

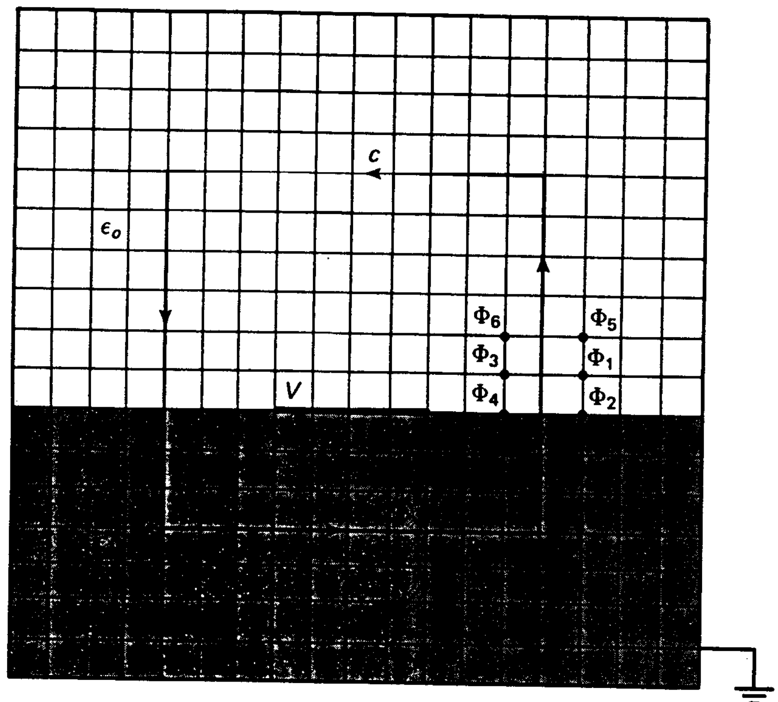

Figure 4.22 Calculation of capacitance in partially filled strip capacitor with dielectric of ϵ_1, using finite difference.

contour c. Evaluating equation 4.57 by using the discrete node values of Φ in Figure 4.22, we obtain

$$\epsilon_o\left(\frac{\Phi_1 - \Phi_3}{2h}\right)h + \epsilon_o\left(\frac{\Phi_5 - \Phi_6}{2h}\right)h + \epsilon_o\left(\frac{\Phi_2 - \Phi_4}{2h}\right)\frac{h}{2} + \epsilon_1\left(\frac{\Phi_2 - \Phi_4}{2h}\right)\frac{h}{2} + \cdots$$

$$\text{(Contributions from other nodes along the closed contour } c) = -q \qquad (4.58)$$

and the capacitance per unit length is

$$C = \frac{q}{V}$$

where V is an initially assumed potential difference between the center conductor and the grounded outer one. This potential is used to solve for Φ, using the finite difference representation of Laplace's equation.

Exercise

For the strip capacitor shown in Figure 4.23, write a computer program to calculate the capacitance per unit length. Input to your program are the dimensions of the strip capacitor, the dielectric constant of the insulation between the two center conductors, and the mesh size h. You will assume that the two center conductors are at the same potential $V = 10$ V (split feed) and the outer conductor is grounded $V = 0$. In choosing

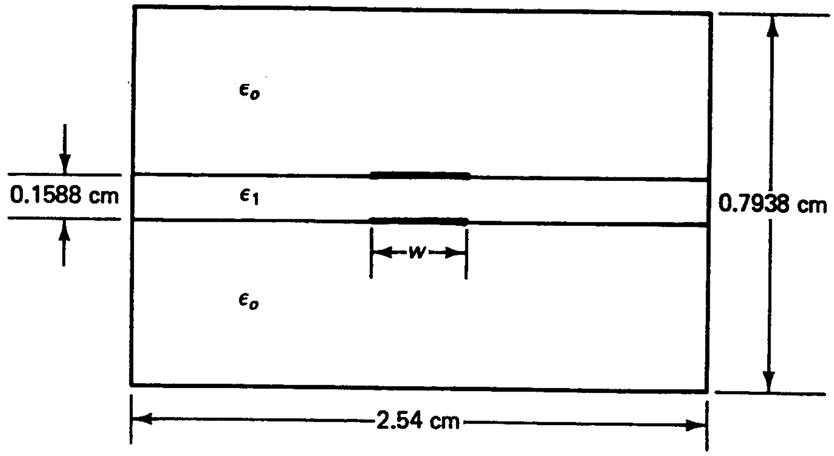

Figure 4.23 Geometry of strip capacitor.

the Gaussian contour to calculate the total charge, it is numerically preferred to take it along the finite difference mesh, midway between the center and outer conductors. After you write your program, you may compare your results with those given in Table 4.2, which illustrates the variation of C with the width of the center strip W and the dielectric constant ϵ_1.

TABLE 4.2 CAPACITANCE PER UNIT LENGTH pF/m FOR VARIOUS VALUES OF W AND ϵ_1

Strip width W (cm)	Teflon $\epsilon_r = 2.05$	Rexolite $\epsilon_r = 2.65$	Air
0.558	58.97	61.83	53.86
0.7145	67.62	70.51	62.34
0.8733	76.77	79.72	71.29

4.9 NUMERICAL SOLUTION OF ELECTROSTATIC PROBLEMS—METHOD OF MOMENTS

In section 4.3, we introduced the concept of electric potential and obtained integral equations that relate the potential to a given charge distribution. For example, if $\rho_s(\mathbf{r}')$ is a surface charge distribution, it is shown that $\Phi(\mathbf{r})$ is given by

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_o}\int_s \frac{\rho_s\,ds'}{|\mathbf{r}-\mathbf{r}'|} \tag{4.59}$$

where $|\mathbf{r}-\mathbf{r}'|$ is the distance from the charge distribution $\mathbf{r}'$ to the point $\mathbf{r}$ at which the potential Φ is to be evaluated. In all the examples we solved to illustrate the application of equation 4.59, we assumed the charge distribution in simple geometries and evaluated equation 4.59 to calculate Φ at specific locations. In many engineering problems, including the determination of capacitance of a system of conductors of complex geometry, the charge distribution is not known and instead the potentials of the