ECE 6340 Homework #6:
Finite Element Method (FEM)

1. Understand the derivation:

   a. Begin with a linear finite element:

   ```
   |_______________|
   Ve1             Ve1
   x1              x2
   ```

   and assume Ve(x) = a + bx. Follow the derivation as we did in class and verify that the elemental shape functions are:

   a1(x) = (x2-x) / (x2-x1)
   a2(x) = (-x1+x) / (x2-x1)

   b. Verify that these satisfy the shape function requirements:
   a1(x1) = a2(x2) = 1.
   a1(x) + a2(x) = 1.

   c. Find the elemental coefficient matrix (Ce):
   Write the matrix equation for the energy in the element,

   $$We = K \bar{Ve}^T \bar{\bar{Ce}} \bar{Ve}$$

   Answer:

   $$\bar{\bar{Ce}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   Specify the constant, K, vector $\bar{Ve}$, and matrix $\bar{\bar{Ce}}$. Specify which values are known, and which are unknown.

   Useful information:

   $$\bar{\bar{C}} = \begin{bmatrix} c11 & c12 \\ c21 & c22 \end{bmatrix} \qquad \bar{\bar{C}}^{-1} = \frac{1}{\det(\bar{\bar{C}})} \begin{bmatrix} c22 & -c12 \\ -c21 & c11 \end{bmatrix}$$

   d. Assemble the global coefficient matrix (C):
   Write the matrix equation for the total energy in this system as a function of x1,x2,x3,x4:

   $$W = K \bar{V}^{-T} \bar{\bar{C}} \bar{V}$$

   ```
   |____|____|____|____|
   x1   x2   x3   x4
   ```

   Answer:

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$$C = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

K = some constant

e. Minimize the total energy in the system:
Take the deriviative of W, and set it equal to zero. This should give you the matrix equation:

$$C\bar{V} = 0$$

f. Given that V1 =1.0, V4 = 0., write the matrix equation you would use to solve for V2 and V3. Solve for them.

g. Given the shape function assumed in part a, plot V(x) from x1 to x4. (This is a piece-wise linear function.)

h. Go back to step a and describe how the size of each of your calculations would have changed if you were using second order linear elements:

```
|________|________|
x1       x2       x3
Ve(x) = a + bx + cx**2
```

How many elements would be used in part d? (Answer: 2)

i. For first and second order elements, give the array sizes which would be required in part d if 100 points were used (x1,x2,…x99).
j. Summarize. Would it be more efficient to use first or second order elements for this problem? Would this generally be the case?
k. List the advantages and disadvantages of the FEM method, as you observe them.

ECE6340 Students
Do problem 6.14 in text.

Grading: (a), (b) 5 points each. All others 10 points.
ECE 6340 Problems 6.14 – 15 points.

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