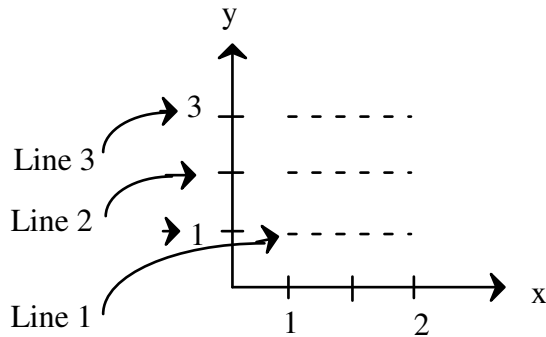


2-D INTEGRATION (TRAPEZOIDAL METHOD)

ECE 5340/6340



$$f(x, y) = xy$$

$$\int_1^2 \int_1^3 (xy) dx dy =$$

 Analytical
6.0

Let

$$n_x = n_y = 2$$

$$h_x = (2 - 1)/2 = 0.5$$

Line 1) $y = 1$

$$f_1(x) = x$$

$$h_y = (3 - 1)/2 = 1$$

$$\begin{aligned} \int_1^2 f_1(x) dx &= \int_1^2 x dx = \left[f_1(1) + 2f_1\left(\frac{3}{2}\right) + f_1(2) \right] \frac{h_x}{2} \\ &= \left[1 + 2\left(\frac{3}{2}\right) + 2 \right] \frac{h_x}{2} = 6 \frac{h_x}{2} = \frac{3}{2} = g(y=1) = g(1) \end{aligned}$$

Line 2) $y = 2$

$$f_2(x) = 2x$$

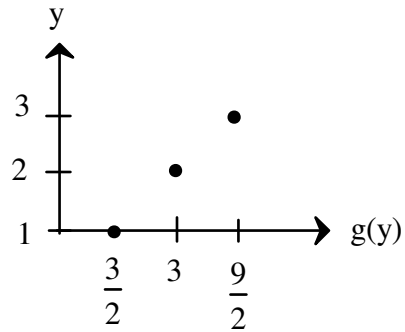
$$\begin{aligned} \int_1^2 f_2(x) dx &= \int_1^2 2x dx = \left[f_2(1) + 2f_2\left(\frac{3}{2}\right) + f_2(2) \right] \frac{h_x}{2} \\ &= \left[2 + 4\left(\frac{3}{2}\right) + 4 \right] \frac{h_x}{2} = 6 h_x = 3 = g(2) \end{aligned}$$

Line 3) $y = 3$

$$f_3(x) = 3x$$

$$\begin{aligned} \int_1^2 f_3(x) dx &= \int_1^2 3x dx = \left[f_3(1) + 2f_3\left(\frac{3}{2}\right) + f_3(2) \right] \\ &= \left[3 + 2\left(\frac{9}{2}\right) + 6 \right] \frac{h_x}{2} = \frac{9}{2} = g(3) \end{aligned}$$

$$\text{ERROR}_x \cong h_x \frac{\partial^2 f}{\partial x^2} = 0$$



$g(y)$:

$$g(1) = 3/2$$

$$g(2) = 3$$

$$G(3) = 9/2$$

$$\int_1^3 g(y) dy = [g(1) + 2g(2) + g(3)] \frac{h_y}{2}$$

$$= \left[\frac{3}{2} + 2(3) + \frac{9}{2} \right] \frac{1}{2} = \boxed{6 \text{ Numerical}}$$

$$\text{ERROR}_y = h_y \frac{\partial^2 f}{\partial y^2} = 0$$

$$\text{ERROR} = \text{ERROR}_x + \text{ERROR}_y = 0$$

Note:

You would have obtained identical results by:

