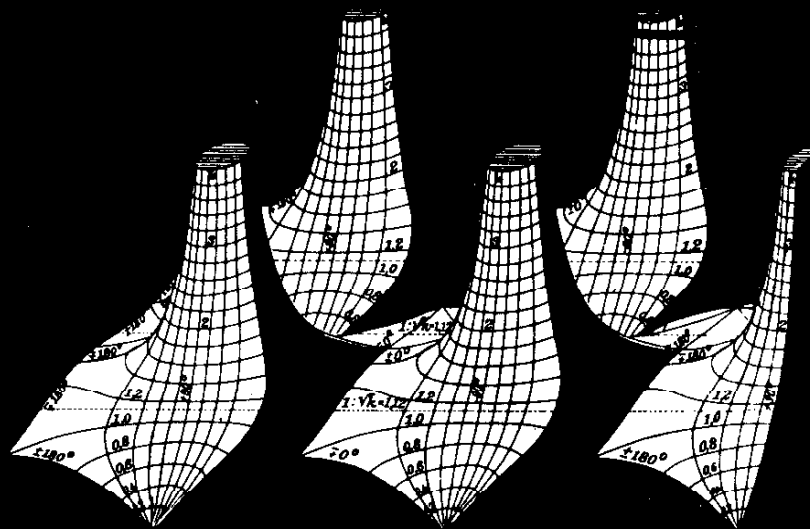




HANDBOOK OF MATHEMATICAL FUNCTIONS

with Formulas, Graphs, and Mathematical Tables

Edited by Milton Abramowitz and Irene A. Stegun

Powers and roots n^k • Common logarithms • Circular sines and cosines for radian arguments • Exponential Integrals $E_n(x)$ • Tetragamma and pentagamma functions • Gamma function for complex arguments • Derivatives of the Legendre Function • Bessel functions—orders 0, 1 and 2, orders 10, 11, 20 and 21, etc. • Spherical Bessel functions • Struve functions • Confluent hypergeometric functions $M(a, b, x)$ • Coulomb wave functions of order zero • Jacobian zeta function $Z(\phi|\alpha)$ • Heuman's lambda function • Table for obtaining periods for invariants g_2 and g_3 • Invariants and values at half-periods • Parabolic cylinder functions • Mathieu functions: characteristic values, joining factors, some critical values • Oblate radial functions—first and second kinds • Sums of reciprocal powers • Bernoulli and Euler numbers • Stirling numbers of the first and second kinds

Table 25.4 ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Abcissas = $\pm x_i$ (Zeros of Legendre Polynomials)			Weight Factors = w_i		
$\pm x_i$		w_i	$\pm x_i$		w_i
$n=2$			$n=8$		
0.57735	0.2691	89626	1.00000	0.0000	0.0000
			0.18343	46424	95650
			0.52553	24099	16329
			0.79666	64774	13627
			0.96028	98564	97536
			0.36268	37833	78362
			0.31370	66450	77007
			0.22238	10344	53374
			0.10122	85362	90376
$n=3$			$n=9$		
0.00000	0.0000	0.0000	0.00000	0.0000	0.0000
0.77459	66692	41483	0.32425	34234	03809
			0.61337	14327	00590
			0.83603	11073	26636
			0.96816	02395	07626
			0.33023	93550	01260
			0.31234	70770	40003
			0.26061	06964	02935
			0.18064	81606	94857
			0.08127	43883	61574
$n=4$			$n=10$		
0.33998	10435	84856	0.14887	43389	81631
0.86113	63115	94053	0.43339	53941	29247
			0.67940	95682	99024
			0.86506	33666	88985
			0.97390	65285	17172
			0.29552	42247	14753
			0.26926	67193	09996
			0.21908	63625	15982
			0.14945	13491	50581
			0.06667	13443	08688
$n=5$			$n=12$		
0.00000	0.0000	0.0000	0.12523	34085	11469
0.53846	93101	05683	0.36783	14989	98180
0.90617	98459	38664	0.58731	79542	86617
			0.76990	26741	94305
			0.90411	72563	10415
			0.98156	06342	46719
			0.24914	70458	13403
			0.23349	25365	38355
			0.20316	74267	23066
			0.16007	83285	43346
			0.10693	93259	95318
			0.04717	53363	86512
$n=6$			$n=16$		
0.23861	91860	83197	0.09501	25098	37637
0.66120	93864	66265	0.28160	35507	79258
0.93246	95142	03152	0.45801	67776	57227
			0.61787	62444	02643
			0.75540	44083	55003
			0.86563	12023	87831
			0.94457	50230	73232
			0.98940	09349	91649
					932596
$n=7$			$n=20$		
0.00000	0.0000	0.0000	0.07652	65211	33497
0.40584	51513	77397	0.22778	58511	41645
0.74153	11855	99394	0.37370	60887	15419
0.94910	79123	42759	0.51086	70019	50827
			0.63605	36807	26515
			0.74633	19064	60150
			0.83911	69718	22218
			0.91223	44282	51325
			0.96397	19272	77913
			0.99312	85991	85094
					924786
$n=8$			$n=24$		
0.18343	46424	95650	0.06405	68928	62605
0.52553	24099	16329	0.19111	88674	73616
0.79666	64774	13627	0.31504	26796	96163
0.96028	98564	97536	0.43379	35076	26045
			0.54542	14713	88839
			0.64809	36519	36975
			0.74012	41915	78554
			0.82000	19859	73902
			0.88641	55270	04401
			0.93827	45520	02732
			0.97472	85559	71309
			0.99518	72199	97021
					360180
			0.12793	81953	46752
			0.12583	74563	46828
			0.12167	04729	27803
			0.11550	56680	53725
			0.10744	42701	15965
			0.09761	86521	04113
			0.08619	01615	31953
			0.07334	64814	11080
			0.05929	85849	15436
			0.04427	74388	17419
			0.02853	13886	28933
			0.01234	12297	99987

Compiled from P. Davis and P. Rabinowitz, Abcissas and weights for Gaussian quadratures of high order, J. Research NBS 56, 35-37, 1956, RP2645; P. Davis and P. Rabinowitz, Additional abcissas and weights for Gaussian quadratures of high order. Values for $n=64, 80$, and 96 , J. Research NBS 60, 613-614, 1958, RP2875; and A. N. Lowan, N. Davids, and A. Levenson, Table of the zeros of the Legendre polynomials of order 1-16 and the weight coefficients for Gauss' mechanical quadrature formula, Bull. Amer. Math. Soc. 48, 739-743, 1942 (with permission).

Table 25.4

ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Abseissas $\pm x_i$ (Zeros of Legendre Polynomials) Weight Factors w_i

$\pm x_i$	$n=32$				w_i
0.04830	76656	87738	316235		0.09654
0.14447	19615	82796	493485		0.09563
0.23928	73622	52137	074545		0.09384
0.33186	86022	82127	649780		0.09117
0.42135	12761	30635	345364		0.08765
0.50689	99089	32229	390024		0.08331
0.58771	57572	40762	329041		0.07819
0.66304	42669	30215	200975		0.07234
0.73218	21187	40289	680387		0.06582
0.79448	37959	67942	406963		0.05868
0.84936	76137	32569	970134		0.05099
0.89632	11557	66052	123965		0.04283
0.93490	60759	37739	689171		0.03427
0.96476	22555	87506	430774		0.02539
0.98561	15115	45268	335400		0.01627
0.99726	38618	49481	563545		0.00701
$\pm x_i$	$n=40$				w_i
0.03877	24175	06050	821933		0.07750
0.11608	40706	75255	208483		0.07703
0.19269	75807	01371	099716		0.07611
0.26815	21850	07253	681141		0.07472
0.34199	40908	25758	473007		0.07288
0.41377	92043	71605	001525		0.07061
0.48307	58016	86178	712909		0.06791
0.54946	71250	95128	202076		0.06480
0.61255	38896	67980	237953		0.06130
0.67195	66846	14179	548379		0.05743
0.72731	82551	89927	103281		0.05322
0.77830	56514	26519	387695		0.04869
0.82461	22308	33311	663196		0.04387
0.86595	95032	12259	503821		0.03878
0.90209	88069	68874	296728		0.03346
0.93281	28082	78676	533361		0.02793
0.95791	68192	13791	655805		0.02224
0.97725	99499	83774	262663		0.01642
0.99072	62386	99457	006453		0.01049
0.99823	77097	10559	200350		0.00452
$\pm x_i$	$n=48$				w_i
0.03238	01709	62869	362033		0.06473
0.09700	46992	09462	698930		0.06446
0.16122	23560	68891	718056		0.06392
0.22476	37903	94689	061225		0.06311
0.28736	24873	55455	576736		0.06203
0.34875	58862	92160	738160		0.06070
0.40868	64819	90716	729916		0.05911
0.46690	29047	50958	404545		0.05727
0.52316	09747	22233	033678		0.05519
0.57722	47260	83972	703818		0.05289
0.62886	73967	76513	623995		0.05035
0.67787	23796	32663	905212		0.04761
0.72403	41309	23814	654674		0.04467
0.76715	90325	15740	339254		0.04154
0.80706	62040	29442	627083		0.03824
0.84358	82616	24393	530711		0.03477
0.87657	20202	74247	885906		0.03116
0.90587	91367	15569	672822		0.02742
0.93138	66907	06554	333114		0.02357
0.95298	77031	60430	860723		0.01961
0.97059	15925	46247	250461		0.01557
0.98412	45837	22826	857745		0.01147
0.99353	01722	66350	757548		0.00732
0.99877	10072	52426	118601		0.00315

Table 25.4

ADSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Abcissas = $\pm x_i$ (Zeros of Legendre Polynomials) Weight Factors = w_i

$\pm x_i$	$n=64$				w_i
0.02435	02926	63424	432509		0.04869
0.07299	31217	87799	039450		0.04857
0.12146	28192	76120	554470		0.04834
0.16964	44204	23992	818037		0.04799
0.21742	36437	40007	084150		0.04754
0.26468	71622	08767	416374		0.04696
0.31132	28719	90210	956158		0.04628
0.35722	01583	37668	115950		0.04549
0.40227	01579	63991	603696		0.04459
0.44636	60172	53464	087985		0.04358
0.48940	31457	07052	957479		0.04247
0.53127	94640	19894	545658		0.04126
0.57189	56462	02634	034284		0.03995
0.61115	53551	72793	250249		0.03855
0.64896	54712	54657	339858		0.03705
0.68523	63130	54233	242564		0.03547
0.71988	18501	71610	826849		0.03380
0.75281	99072	60531	896612		0.03205
0.78397	23589	43341	407610		0.03023
0.81326	53151	22797	559742		0.02833
0.84062	92962	52580	362752		0.02637
0.86599	93981	54092	819761		0.02435
0.88931	54459	95114	105853		0.02227
0.91052	21370	78502	805756		0.02013
0.92956	91721	31939	575821		0.01795
0.94641	13748	58402	816062		0.01572
0.96100	87996	52053	718919		0.01346
0.97332	68277	89910	963742		0.01116
0.98333	62538	84625	956931		0.00884
0.99101	33714	76744	320739		0.00650
0.99634	01167	71955	279347		0.00414
0.99930	50417	35772	139457		0.00178
$n=80$					
0.01951	13832	56793	997654		0.03901
0.05850	44371	52420	668629		0.03895
0.09740	83984	41584	599063		0.03883
0.13616	40228	09143	886559		0.03866
0.17471	22918	32646	812559		0.03842
0.21299	45028	57666	132572		0.03812
0.25095	23583	92272	120493		0.03777
0.28852	80548	84511	853109		0.03736
0.32566	43707	47701	914619		0.03689
0.36230	47534	99487	315619		0.03637
0.39839	34058	81969	227024		0.03579
0.43387	53708	31756	093062		0.03516
0.46869	66151	70544	477036		0.03447
0.50280	41118	80704	987594		0.03373
0.53614	59208	97131	932020		0.03294
0.56867	12681	22709	784725		0.03210
0.60033	06228	29751	743155		0.03121
0.63107	57730	46871	966248		0.03027
0.66085	98989	86119	801736		0.02928
0.68963	76443	42027	600771		0.02825
0.71736	51853	62099	880254		0.02718
0.74400	02975	83597	272317		0.02607
0.76950	24201	35041	373866		0.02492
0.79383	27175	04605	449949		0.02373
0.81695	41386	81463	470371		0.02250
0.83883	14735	80255	275617		0.02124
0.85943	14066	63111	096977		0.01995
0.87872	25676	78213	828704		0.01862
0.89667	55794	38770	683194		0.01727
0.91326	31025	71757	654165		0.01589
0.92845	98771	72445	795953		0.01449
0.94224	27613	09872	674752		0.01306
0.95459	07663	43634	905493		0.01162
0.96548	50890	43799	251452		0.01016
0.97490	91405	85727	793386		0.00868
0.98284	85727	38629	070418		0.00719
0.98929	13024	99755	531027		0.00569
0.99422	75409	65688	277892		0.00418
0.99764	98643	98237	688900		0.00266
0.99955	38226	51630	629880		0.00114
					49500
					03186
					941534

Table 25.4

ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Abcissas = $\pm x_i$ (Zeros of Legendre Polynomials)				Weight Factors = w_i			
$\pm x_i$				w_i			
$n = 96$							
0.01627	67448	49602	969579	0.03255	06144	92363	166242
0.04881	29851	36049	731112	0.03251	61187	13868	835987
0.08129	74954	64425	558994	0.03244	71637	14064	269364
0.11369	58501	10665	920911	0.03234	38225	68575	928429
0.14597	37146	54896	941989	0.03220	62047	94030	250669
0.17809	68823	67618	602759	0.03203	44562	31992	663218
0.21003	13104	60567	203603	0.03182	87588	94411	006535
0.24174	31561	63840	012328	0.03158	93307	70727	168558
0.27319	88125	91049	141487	0.03131	64255	96861	355813
0.30436	49443	54496	353024	0.03101	03325	86313	837423
0.33520	85228	92625	422616	0.03067	13761	23669	149014
0.36569	68614	72313	635031	0.03029	99154	20827	593794
0.39579	76498	28908	603285	0.02989	63441	36328	385984
0.42547	89884	07300	545365	0.02946	10899	58167	905970
0.45470	94221	67743	008636	0.02899	46141	50555	236543
0.48345	79739	20596	359768	0.02849	74110	65085	385646
0.51169	41771	54667	673586	0.02797	00076	16848	334440
0.53938	81083	24357	436227	0.02741	29627	26029	242823
0.56651	04185	61397	168404	0.02682	68667	25591	762198
0.59303	23647	77572	080684	0.02621	23407	35672	413913
0.61892	58401	25468	570386	0.02557	00360	05349	361499
0.64416	34037	84967	106798	0.02490	06332	22483	610288
0.66871	83100	43916	153953	0.02420	48417	92364	691282
0.69256	45366	42171	561344	0.02348	33990	85926	219842
0.71567	68123	48967	626225	0.02273	70696	58329	374001
0.73803	06437	44400	132851	0.02196	66444	38744	349195
0.75960	23411	76647	498703	0.02117	29398	92191	298988
0.78036	90438	67433	217604	0.02035	67971	54333	324595
0.80030	87441	39140	817229	0.01951	90811	40145	022410
0.81940	03107	37931	675539	0.01866	06796	27411	467385
0.83762	35112	28187	121494	0.01778	25023	16045	260838
0.85495	90334	34601	455463	0.01688	54798	64245	172450
0.87138	85059	09296	502874	0.01597	05629	02562	291381
0.88689	45174	02420	416057	0.01503	87210	26994	938006
0.90146	06353	15852	341319	0.01409	09417	72314	860916
0.91507	14231	20898	074206	0.01312	82295	66961	572637
0.92771	24567	22308	690965	0.01215	16046	71088	319635
0.93937	03397	52755	216932	0.01116	21020	99838	498591
0.95003	27177	84437	635756	0.01016	07705	35008	415758
0.95968	82914	48742	539300	0.00914	86712	30783	386633
0.96832	68284	63264	212174	0.00812	68769	25698	759217
0.97593	91745	85136	466453	0.00709	64707	91153	865269
0.98251	72635	63014	677447	0.00605	85455	04235	961683
0.98805	41263	29623	799481	0.00501	42027	42927	517693
0.99254	39003	23762	624572	0.00396	45543	38444	686674
0.99598	18429	87209	290650	0.00291	07318	17934	946408
0.99836	43758	63181	677724	0.00185	39607	88946	921732
0.99968	95038	83230	766828	0.00079	67920	65552	012429