University of Utah
Department of Electrical Engineering

Gaussian Quadrature Formula: an Implementation

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GAUSS QUADRATURE FORMULA: AN IMPLEMENTATION

```
C  --- Program for computing Complex Integrals using Gauss        ---
C  --- method (with 9 points).                                    ---
C  --- Developed by Dr.Gianluca Lazzi, 1995                       ---

      program Integral


      implicit none
C
C  --- We use Gaussian integration with 'm'=9 points:             ---
C  --- Abscissas are stored in the vector 'x';                    ---
C  --- Coefficients are stored in the vector 'h'.                 ---
C
      integer m
      parameter(m=9)
      real*8 h(m),x(m)
C
C  --- Remind that abscissas are symmetric respect x=0 and        ---
C  --- are defined for the range -1<x<1.                          ---
C  --- If you want to improve the quadrature formula, you         ---
C  --- must increase the number of points m.                      ---
C  --- You can find abscissas and weighting factors in the        ---
C  --- book : Abramowitz - Stegun, "Handbook of                   ---
C  --- Mathematical Functions".                                   ---
C
      data h(1),h(2),h(3),h(4)/0.0812743883,0.1806481606,0.2606106964,
     ,0.3123470770/
      data h(5),h(6),h(7),h(8),h(9)/0.3302393550,0.3123470770,
     ,0.2606106964,0.1806481606,0.0812743883/
      data x(1),x(2),x(3),x(4)/-0.9681602395,-0.8360311073,
     ,-0.6133714327,-0.3242534234/
      data x(5),x(6),x(7),x(8),x(9)/0.,0.3242534234,0.6133714327,
     ,0.8360311073,0.9681602395/
C
C  --- 'estr1' and 'estr2'are, respectively, left and right       ---
C  ---     extreme of the integral.                               ---
C  --- 'prec' is the relative requested precision :               ---
C  ---   handle this value carefully. The result is very          ---
C  ---   accurate from the first step; so, sometimes, it is       ---
C  ---   impossible to obtain the requested precision because     ---
C  ---   this is computed by the relative difference              ---
C  ---   between previous and actual value of the integral.       ---
C  ---   The program will stop if a maximum number of             ---
C  ---   intervals is reached. In this case you probably          ---
C  ---   obtain the highest accuracy.                             ---
C  --- 'monoi' is the function that compute the integral.         ---
C  --- 'integrand' is the integrand function: it may be           ---
C  ---   complex.                                                 ---
C  --- 'result' will contain the result of the integration.       ---
C
      real*8 estr1,estr2,prec

      parameter(estr1=-10 ,estr2-10 ,prec=1.E-6)

      complex*16 monoi, integrand, result
```

```
      external integrand
C
C  --- Begin of Main Program ---
C
      result=monoi(integrand,h,x,estr1,estr2,m,prec)
      write(6,*) 'Result=',result

      stop
      end
```

```
C
C  --- Routine for computing monodimensional complex integrals    ---
C
C  --- 'itg' is the integrand.                                    ---
C  --- 'int' is the actual value of the integral.                 ---
C  --- 'itg' is the value of the integral at the previous step.   ---
C  --- 'n' is the number of intervals in which is sub-divided     ---
C  ---    the interval estr2-estr1.                               ---
C  --- 'support' is the abscissa for the integrand.               ---

      complex*16 function monoi(itg,h,x,estr1,estr2,m,prec)
      complex*16 itg,int,itp,support
      integer m,n,con1,con2
      real*8 h(m),x(m),prec,estr1,estr2
      external itg
C
      int=(0.,0.)
      itp=(0.,0.)
      itp=(0.,0.)

C --- Start with 16 intervals (9 points for each one) ...   ---

      n=16

C --- ... and now start the cycle !                          ---

10    do 20,con1=1,n
      do 20,con2=1,m
        support=(((estr2-estr1)/(2.*n))*(x(con2)+2.*con1-1)+estr1)
        int=int+h(con2)*itg(support)
20    continue
      int=int*((estr2-estr1)/(2.*n))
      write(6,*) 'Number of intervals:',n

C --- If the relative precision is not reached, or this is  ---
C ---   the first step, and the number of intervals is not  ---
C ---   greater than 65536 (our precision limit), we        ---
C ---   increase the number of intervals and restart the    ---
C ---   cycle.                                              ---

      if ((((abs(int-itp)).gt.(prec*abs(int))).or.(n.eq.16)).
     .and.(n.le.65536)) then
          itp=int
          n=n*2
          int=(0.,0.)
```

```
            goto 10
      end if
      monoi=int
      return
      end


C
C --- Definition of Integrand                                          ---
C --- Note that the integrand may be a complex function                ---
C ---    of complex variable!                                          ---
C
      complex*16 function integrand(x)
      complex*16 x
      integrand=x
      return
      end
```

GAUSSIAN QUADRATURE FORMULA: RESULTS

To test the proposed routine, results for several integrals are presented in the following.

Integral	*Exact Value*	*Program Result*
$\int_{-10}^{10} x\,dx$	0	8.59E-15
$\int_{-10}^{10} x^2\,dx$	666.666666	666.666666
$\int_{-10}^{10} x^3\,dx$	0	4.97E-14
$\int_{-10}^{10} x^4\,dx$	40000	39999.999999

Furthermore, good results are obtained also for integrals with infinite extremes. Obviously, in this case we must choice appropriate finite extremes by a previous analytical study of the integrand. For example:

$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)} dx = 1 \approx \frac{1}{2\pi}\int_{-10000}^{+10000} \frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)} dx$, for which we obtain, by the program, the result 0.99998.

For the integral

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)} e^{j2x} dx = 0,$$

we obtain by the program the following results:

$$\frac{1}{2\pi}\int_{-10000}^{+10000}\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}e^{j2x}dx = -1.845E\text{-}5 - j1.28E\text{-}15;$$

$$\frac{1}{2\pi}\int_{-50000}^{+50000}\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}e^{j2x}dx = -1.36E\text{-}6 - j1.04E\text{-}14;$$

$$\frac{1}{2\pi}\int_{-100000}^{+100000}\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}e^{j2x}dx = 2.27E\text{-}7 - j\ 2.568E\text{-}15.$$

Observe that the last types of integrals are Fourier-type integrals. So, for example, it is possible to evaluate Fourier-transforms and inverse Fourier-transforms.