ECE 6340 HOMEWORK 5: METHOD OF MOMENTS

Solve for the charge distribution on a wire using method of moments, Galerkin method, with triangular basis functions.

I. Theory:

(Follow the steps below. Describe the integrals, but do not program them. No programming required.)

a) Write the equation for the triangular basis function, $u_n(x)$
b) For the Galerkin method, what is the weighting function?
c) Beginning with the integral equation:

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_o}\int_V \frac{\rho(\bar{r}')}{|\bar{r}-\bar{r}'|}dv' \qquad (1)$$

Clearly indicate what parts are the unknown function, U(x'), the kernal, g(x,x'), and the forcing function f(x) so that

$$F(\bar{x}) = \int_{x=0}^{L} U(x')g(x,x')dx' \qquad (2)$$

Remember that we're actually solving the integral in one-dimension, so replace all of the r's and r-primes with x's.

d) V(x) is assumed to be constant on the wire. Why?
e) Rewrite the unknown function U(x) as a sum of basis functions. Describe what this means. Suppose that when you finish your MoM simulation using 4 points, you obtain the results that p(x1)=1.0, p(x2)=2.0, p(x3) = 2.0, p(x4) = 1.0. Draw the charge distribution on the wire.
f) Rewrite (2) using this new representation of U(x). Describe exactly what integral needs to be done. How would you do this integral (numerically or analytically)? How many equations and how many unknowns does this give you?
g) Apply the weighting functions to the integral in (f). (Take the inner product.) Describe exactly what integral needs to be done and how you would do it. How many equations in how many unknowns do you have now?
h) Write the equations from (g) in terms of a matrix equation Ax=b and specify exactly what the terms A_{11} (the top left hand element of the matrix) and A_{12} (the element directly to its right) of this matrix would be. Give the sizes of all elements in the matrix equation.
i) When you solve this matrix equation, what do you get?
j) How can you find V(x,y) at some location far from the wire?
k) How can you find E(x,y) at some location far from the wire?

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l) From your knowledge of V on the wire, and your answer to k, prove that the tangential electric field on the metal wire is zero.
m) Summarize. What are the advantages and disadvantages of the method of moments?

Possible Final Project
Repeat the analysis of the wire that you did in the FDFD project using Method of Moments. See text section 5.4

II. **Programming**:

Write a program which will solve for the charge distribution on a straight wire. From this charge distribution, solve for the potential (V) at any location on or off of the wire.
Make your program general:
1) Allow any basis or weighting function by defining these as functions.
2) Allow the user to input the length and diameter of the wire.

Hand in:
[ ] **All calculations from Part I.**

A simple handwritten answer will be sufficient for most sections. Typed answers are even better if you don't mind taking the extra time for the equations. A title page or table of contents is not necessary, as long as each problem is clearly labeled and easy to follow. Remember your goal is to demonstrate to me that you understand what is going on. The messier your work is, the more inclined I am to scrutinize it and look for reasons to dock points.

[ ] **Source code, well commented**

Remember that your code serves two purposes. 1: It shows me that you did your own work, and 2: You may need to look it up someday. There is a good chance that perhaps 1/3 of you will need to someday come back and consult the knowledge gained from this course. So take care to neatly write your code. It's for your own benefit.

[ ] **Plots of charge distributions:**
a) Length of rod = 1m, radius = 1mm, rectangular basis function, dirac delta weighting function
b) Length of rod = 1m, radius = 10mm, rectangular basis function, dirac delta weighting function
c) Length of rod = 2m, radius = 1mm, rectangular basis function, dirac delta weighting function
d) Length of rod = 1m, radius = 1mm, rectangular basis function, rectangular weighting function.

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e) (ECE6340 students only) Length of rod = 1m, radius = 1mm, triangular basis function, rectangular weighting function. Hint: You will have to either repeat the self-term for multiple elements, or skip the points where 1/|x-x'| = 0.

DO NOT just print out graphs and turn them in. Clearly denote each plot with an explanation of what it represents. Label it with the appropriate problem letter. Also place a label somewhere on the page that indicates the length of the rod, the radius, the basis functions, etc.

Remember to properly label your axes with CORRECT UNITS. A plot without units has no physical meaning.

Include a brief discussion about each plot (2-3 sentences). Either place this on the same page as the plot itself, or include this with the conclusion section. Compare and contrast the graphs with each other. Note their differences and similarities. What differences are the basis functions making? Why is this important? Things like that. It is not enough to just go through the motions. Show me that you understand.

[ ] **Conclusion!** Discuss what you did, what you learned, where the trouble was, and what important insight you gained. Just a simple paragraph or two.

Grading:

Part I	3 points (approx) for each (a) – (m)	______ / 45
Part II		______ / 55
	Total	______ / 100

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