Method of Moments – Method of Weighted Residuals

- Special Case: Galerkin Method (also developed via FE)
- Integral Equation Method
- Ideal for:
  - wires
  - impenetrable (metal) objects (surface patch method)
  - Large, simple scatterers (half-plane, layered plane, etc.) where Green's function can be derived. Then small perturbations can be added
- Not ideal for:
  - Complex, penetrable objects (too much memory required)

<u>General Method:</u>

Given a function:

$$f(\bar{r}) = \int_v U(\bar{r}')g(\bar{r},\bar{r}')dv' \tag{1}$$

Unknown: U(r')
Known (Green's Function): g(r,r')
Forcing function: f(r)
(forcing function is NOT the source, it is better to call it the "testing" function, because it is where the effect of the source is tested.)
observation point: r (vector to observation point)
source point: r'

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Forexample:

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_o}\int_v \rho(\bar{r}')\frac{1}{|\bar{r}-\bar{r}'|}dv' \qquad Electrostatic \quad ch\arg es$$

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_v J_v(\bar{r}')\frac{e^{-jkR}R}{R}dv' \qquad Scattering\ \Pr oblems$$

$$\bar{B} = \nabla X \bar{A} --> \bar{H}$$

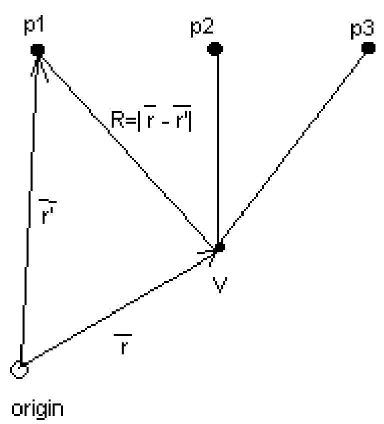
$$R = |\bar{r}-\bar{r}'|$$

$$\bar{E} = -j\omega\bar{A} - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla \bullet \bar{A})$$

$$E_z(\bar{r}) = \frac{-k\eta}{4}\int_S J_x(\bar{\rho}')H_o^{(2)}(k|\bar{\rho}-\bar{\rho}'|)dS' \qquad 2D\_Scattering$$

Example:

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_o}\int_v \rho(\bar{r}')\frac{1}{|\bar{r}-\bar{r}'|}dv$$



Write the unknown U(r’) as a set of basis functions:

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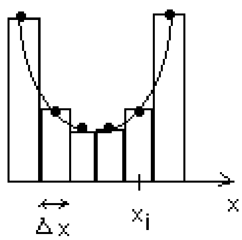
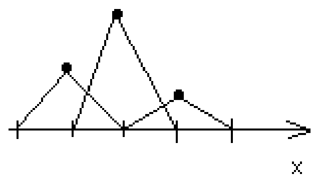


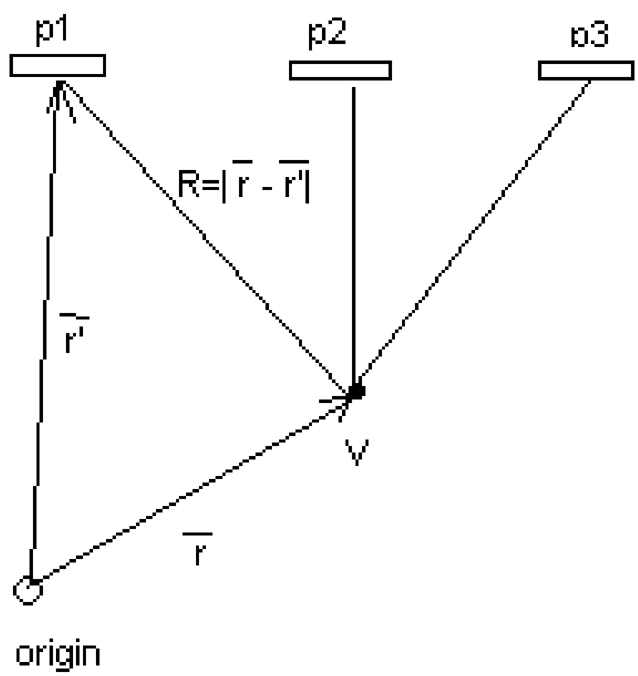
Pulse Basis
Function
(Point Matching)



Triangular Basis Function
(continuous first
derivative)

Our example:

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_v \rho(\bar{r}')\frac{1}{|\bar{r}-\bar{r}'|}dv$$



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$$U(\overline{r'}) = \sum_{n=1}^{N} u_n(\overline{r'}) A_n$$

$$u_n(\overline{r'}) = basis\_function$$

$$A_n = amplitude\_(weight!)$$

$$Then\_(1)\_becomes:$$

$$f(\overline{r}) = \int_v \left(\sum_{n=1}^{N} u_n(\overline{r'}) A_n \right) g(\overline{r}, \overline{r'}) dv'$$

$$OR:$$

$$f(\overline{r}) = \sum_{n=1}^{N} A_n \left(\int_v u_n(\overline{r'}) g(\overline{r}, \overline{r'}) dv' \right)$$

$$f(\overline{r}) = \sum_{n=1}^{N} A_n g_n(\overline{r})$$

Integral above usually evaluated numerically.
Now we have 1 equation and N unknowns (An).

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$\mathrm{Re}\,sidual:$

$$R(\bar{r}) = f(\bar{r}) - \sum_{n=1}^{N} A_n g_n(\bar{r}) = 0$$

$"Weight"\_the\_residual:$

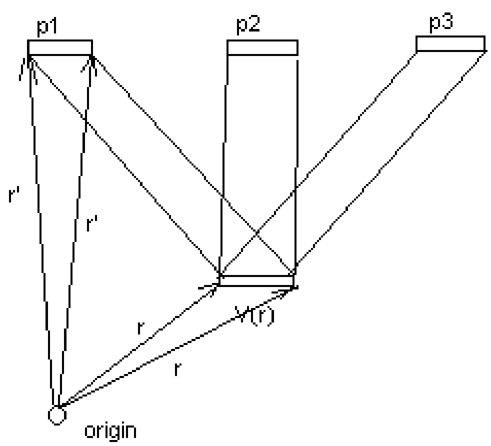
$$\iint_S W_m R(\bar{r})dS \equiv \langle W_m, R(\bar{r}) \rangle = 0 \qquad Inner\_product$$

$Weight\_function\_may\_be\_dirac, rect, triangle, etc.$

$$\left\langle W_m, \sum_{n=1}^{N} A_n g_n(\bar{r}) \right\rangle = \langle W_m, f(\bar{r}) \rangle$$

$$\sum_{n=1}^{N} A_n \langle W_m, g_n(\bar{r}) \rangle = \langle W_m, f(\bar{r}) \rangle$$

Our example:



In order to get N equations for N unknowns, we need to sample V(r) at N locations. Then we can write a matrix:

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$$\begin{bmatrix} \langle W_1, g_1 \rangle & \langle W_1, g_2 \rangle & & \langle W_1, g_N \rangle \\ \langle W_2, g_1 \rangle & \langle W_2, g_2 \rangle & & \\ & & & \\ & & & \langle W_N, g_N \rangle \end{bmatrix} \begin{bmatrix} A_1 \\ \\ \\ A_N \end{bmatrix} = \begin{bmatrix} \langle W_1, f \rangle \\ \\ \\ \langle W_N, f \rangle \end{bmatrix}$$

$$\langle W_m, g_n \rangle = \iint_S W_m(\bar{r}) \bullet g_n(\bar{r}) dS$$

$$= \underbrace{\iint_S \underbrace{W_m(\bar{r})}_{(3)\,Choose\_weight\_fn} \bullet \underbrace{\left\{ \int_V \underbrace{u_n(\bar{r})}_{(1)\,Choose\_basis\_fn} g(\bar{r}, \bar{r}') dV' \right\}}_{(2)\,gn(r)} dS}_{(4)\,do\_\text{int}\,egral\_numerically}$$

$$\langle W_m, f \rangle = \iint_S W_m(\bar{r}) \bullet f(\bar{r}) dS$$

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