

Why the DFT is Faster Than the FFT for FDTD Time-to-Frequency Domain Conversions

C. M. Furse, *Member, IEEE*, and O. P. Gandhi, *Fellow, IEEE*

Abstract—Although it is a time-domain method, the finite-difference time-domain (FDTD) method has been used extensively for calculating frequency domain parameters such as specific absorption rate, radar cross-section, and S -parameters. When a broad frequency band is of interest, using a broad-band pulsed excitation can provide this frequency response with a single FDTD simulation. The frequency domain data can be calculated from the time domain data using either a discrete Fourier transform (DFT) or a fast Fourier transform (FFT). This letter examines both methods and analyzes why the DFT is generally more efficient and easier to use than the FFT for FDTD time-to-frequency domain conversions.

I. INTRODUCTION

ALTHOUGH IT is a time-domain method, the finite-difference time-domain method has been used extensively to calculate frequency-domain parameters. These include specific absorption rate (SAR) [1], radar cross-section [2], current distribution [3], and S -parameters [4]. If a broad frequency range is of interest, a broad-band pulsed excitation can be used to obtain the frequency response from a single FDTD simulation. This requires conversion of the original time domain data to the frequency domain, which is done with either the discrete Fourier transform (DFT) [5], [6] or the fast Fourier transform (FFT) [7], [8]. Although the FFT is commonly thought to be "faster," it is actually significantly "slower" than the DFT for most FDTD simulations, requires much more memory, and has limitations on the resolution of the calculated frequencies. Since many applications, such as calculation of SAR or RCS, require a huge number of time-to-frequency domain conversions, choosing the most efficient method is very important. This paper compares the efficiency and use of the DFT and the FFT for FDTD simulations and demonstrates why the DFT is generally more efficient and more accurate than the FFT for these applications.

II. FINDING THE FREQUENCY RESPONSE FROM A PULSED FDTD SIMULATION USING FOURIER TRANSFORM METHODS

In order to obtain broad-band frequency domain data from an FDTD simulation, a broad-band pulse is used as the incident field. The time domain data is then converted to the frequency domain using either the DFT or the FFT.

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The authors are with the Department of Electrical Engineering, University of Utah, Salt Lake City, UT 84112 USA.

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The DFT series summation is given by

$$G(m\Delta f) = \Delta t \sum_{n=0}^{N_{\text{DFT}}-1} g(n\Delta t) \exp \left[\frac{-j2\pi mn}{N_{\text{DFT}}} \right], \quad m = 0, 1, 2, \dots, N_{\text{DFT}} - 1 \quad (1)$$

where

$G(m\Delta f)$ is the complex value of the magnitude and phase of the equivalent steady-state sine wave at frequency $m\Delta f$

$g(n\Delta t)$ is the time-domain value of the pulse at time $n\Delta t$

Δf is the frequency resolution of the frequency-domain calculations

m is the frequency index, $m = 0, 1, 2, \dots, N_{\text{DFT}} - 1$

Δt is the sampling period of the DFT

N_{DFT} is the length of the DFT summation $= 1/(\Delta f \Delta t)$.

This summation is updated at every FDTD time step, so storage of the complex value, $G(m\Delta f)$, is required. Since commonly used pulse shapes do not have constant frequency responses, the final values must also be normalized by the DFT of the incident pulse to obtain frequency domain data equivalent to data which would have been obtained if the model had been illuminated by a 1 V/m incident sine wave at each frequency of interest.

For the FFT, the complete time history of the values at all points of interest must be stored and entered into an FFT program after the FDTD simulation has been completed. When a large number of time-to-frequency domain transformations are made, this can require a huge amount of disk storage. As with the DFT, the final calculated values must also be normalized by the frequency response of the incident pulse.

III. COMPARISON OF THE DFT AND FFT METHODS

Since the DFT and FFT are both based on the summation in (1), their theoretical accuracy is virtually identical. In practice, however, the accuracy of the FFT is often compromised by the limitation on N_{DFT} , which is necessary to provide its efficiency. The FFT is generally known to be a highly efficient method, far surpassing the DFT in computational efficiency. This does not hold true for many FDTD applications, however, where the DFT is generally as, or more, computationally efficient than the FFT and requires far less computer storage. This occurs because of several basic assumptions unique to FDTD time-to-frequency-domain conversions.

First, the sampling period of the FDTD simulation is much smaller than that required by the Nyquist rate [9]. Specifically, suppose that the cell size is taken to be the nominal value of $\Delta x = \lambda_{\text{min}}/10 = (c_0/F_{\text{max FDTD}})/10$. The time resolution

is chosen to be $\Delta t = \Delta x / (2c_0)$ for stability of the FDTD algorithm [10, p. 297]. This gives $\Delta t = 1 / (20 * F_{\text{max FDTD}})$, which is 20 samples per period, compared with two samples per period required by the Nyquist criterion. Without desampling, this means that the FFT will waste time computing extraneous frequencies up to 10 times those properly modeled by the FDTD simulation. The DFT can be used to calculate only the properly modeled frequencies, thus eliminating this waste.

Desampling the FDTD time domain data improves the efficiency of both the DFT and the FFT and eliminates the waste of calculating frequencies above the range of the FDTD simulation. For a cell size, $\Delta x = \lambda_{\text{min}}/10$, which gives a sampling rate of 20 samples per period, for instance, only every 10th time step of the simulation is required for the Fourier transform. Then, the maximum frequency that will be calculated by the Fourier transform is equal to the maximum frequency of the FDTD simulation.

A second reason that the DFT becomes more efficient than the FFT for pulsed FDTD algorithms is that the number of FDTD time steps to reach convergence, N_{FDTD} , is generally far less than the length of the Fourier transform, N_{DFT} . This happens particularly if the frequency resolution, Δf , is chosen to be small, as it often is to allow calculation at a specific frequency. Suppose, for instance, that information at the frequencies 40, 350, and 915 MHz is of interest. To obtain all three of these frequencies from a single FDTD run, Δf must be chosen to be 5 MHz, to evenly factor all three frequencies. For the 1.31 cm resolution man model [11], for instance, $\Delta t = 21.83$ picoseconds, so $N_{\text{DFT}} = 9160$, and this model generally converges in less than 1000 time steps. This means that all fields are effectively zero after 1000 time steps, so the DFT summation does not need to be calculated for the time steps 1001–9160, which is a significant savings.

If desampling is used with a desampled time resolution of $\Delta t' = 10 \Delta t$, the length of the Fourier transform becomes $N_{\text{DFT}} = 916$. The FFT must be calculated for this full length, but the DFT summation can be stopped $n = 100$ ($N_{\text{FDTD}} = 1000$) using desampling. This again gives a significant savings.

It is worth noting that the ability to choose any Δf , and therefore compute values for any specific frequency, is a significant advantage of the DFT over the FFT. The FFT algorithm gains its efficiency by having N_{DFT} be a power of some integer. Specifically, the highly efficient radix-2 FFT algorithm [12] requires that $N_{\text{DFT}} = 2^n$, that significantly limits its options, and hence the available Δf values and the specific discrete frequencies which may be calculated. For the man model case, using $N_{\text{DFT}} = 8192$ ($n = 13$), for instance, 8192 frequencies would have to be calculated to obtain values for 39.13, 301.86, and 916.76 MHz, approximating the desired frequencies of 40, 300, and 915 MHz.

The computational requirements of the DFT and FFT can be compared by examining the number of complex multiplications required for each method. The radix-2 FFT algorithm requires $(N_{\text{FFT}}/2) \log_2(N_{\text{FFT}})$ complex multiplications, compared to $N_{\text{FDTD}}N_f$ for the DFT algorithm using the savings described above, where

N_{FFT} is the number of frequencies computed using the FFT algorithm

N_{FDTD} is the number of time steps for the FDTD simulation (or the number of terms in the DFT summation if desampling is used)

N_f is the number of frequencies of interest computed by DFT.

For the 1.31-cm resolution man model test case described above, with $\Delta t = 21.83$ ps, and $\Delta f = 5$ MHz, suppose $N_{\text{FFT}} = 8192$, $N_{\text{FDTD}} = 1000$, and $N_f = 3$. Then, the number of complex multiplications without desampling is 53 248 for the FFT algorithm and 3000 for the DFT algorithm. Using a desampled time resolution of $\Delta t' = 218.3$ ps and $N_{\text{FFT}} = 1024$ requires and 5120 complex multiplications for the FFT and 300 for the DFT. Desampling significantly reduces the computational time of both methods, but their relative efficiency remains about the same, with the DFT being significantly more efficient than the FFT.

To further emphasize the significance of this savings, suppose that this was used to compute the SAR distribution at every point in the 1.31 cm resolution man model (404,838 locations). This requires a Fourier transform of the E_x , E_y , and E_z components at every location. The FDTD algorithm requires two real multiplications (conservatively equivalent to one complex multiplication) for each of six field components for 1000 time steps, which is 2.4×10^9 complex multiplications. Using desampling, the DFT would require 0.36×10^9 complex multiplications (1/7 of the FDTD simulation), or the FFT would require 6.22×10^9 complex multiplications (three times as much as the FDTD simulation itself!). Without desampling, the DFT would require 3.6×10^9 complex multiplications, and the FFT would require 64.7×10^9 complex multiplications.

The exact efficiency comparison between DFT and FFT depends on the frequency resolution, Δf , which controls N_{FFT} , the number of time steps to reach convergence, and the number of frequencies of interest. As Δf decreases, the DFT becomes relatively more efficient.

Another advantage of the DFT algorithm is that the time-history of the fields does not need to be stored, because the summation is updated at each time step of the FDTD simulation. For the FFT, on the other hand, the complete time-history must be stored at each location. For the 1.31-cm resolution man model using desampling, this means that 100 time steps must be stored for 404 838 locations, which is over 10 times the storage requirements for the FDTD simulation itself. If not stored in core memory, the burden falls on disk storage, which is also often limited. For many FDTD cases, the DFT will be as or more computationally efficient than the FFT for time-to-frequency-domain conversions, requires far less additional storage, and has the added advantage that the frequency resolution can be chosen to allow precise calculation of desired frequencies.

IV. CONCLUSION

The relative merits of the DFT and FFT for FDTD time-to-frequency domain conversions are summarized in Table I, shown on the next page, for the test case of SAR calculations for the 1.31-cm resolution man model. For this and other

TABLE I
COMPARISON OF DFT AND FFT FOR TIME-TO-FREQUENCY CONVERSIONS REQUIRED TO COMPUTE
SAR DISTRIBUTION IN 1.31-cm RESOLUTION MAN MODEL

	DFT	FFT	FDTD
Number of Complex Multiplications (w/o desampling)	3.6×10^9	64.7×10^9	2.4×10^9
Number of Complex Multiplications (w/ desampling)	0.36×10^9	6.23×10^9	2.4×10^9
Storage Requirement (w/o desampling)	0.8 MBytes	404.8 Mbytes	≈ 2.83 MBytes
Storage Requirement (w/ desampling)	0.8 MBytes	40.48 Mbytes	≈ 2.83 MBytes
Choice of Frequency	unlimited	limited by radix-2 algorithm	

FDTD applications, the DFT is more computationally efficient than the FFT and also requires less memory. For applications requiring a large number of these conversions, significant savings in computer time and memory can be realized by using the DFT instead of the FFT.

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