ECE 5340 / 6340
Finite Difference Time Domain (FDTD)
Introduction

Maxwell's Equations (time domain, differential form):

$$\nabla X \overline{E} = -\mu \frac{\partial \overline{H}}{\partial t} (no\ magnetic\ loss)$$

$$\nabla X \overline{H} = \sigma \overline{E} + \varepsilon \frac{\partial \overline{E}}{\partial t}$$

When you write these equations out, you will have equations that look like:
A **x** + B **y** + C **z** = D **x** + E **y** + F **z**
The "x" vector terms must therefore be equal (A = D), the "y" vector terms must be equal (B=E), and the "z" vector terms must be equal (C=F).

For Maxwell's equations equate Vector components.
These are the "x" terms.

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right)$$

$$\frac{\partial E_y}{\partial t}\ and\ \frac{\partial E_z}{\partial t}\ similar$$

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right)$$

$$\frac{\partial H_y}{\partial t}\ and\ \frac{\partial H_z}{\partial t}\ similar$$

Discretize Space (Δx):
This format is called a "Yee" cell, named after Kane Yee who developed this method in 1966. Note that the E,H components are all located at different locations, ½ cell apart. This is called "leap-frogging" and is very useful when taking the central differences.

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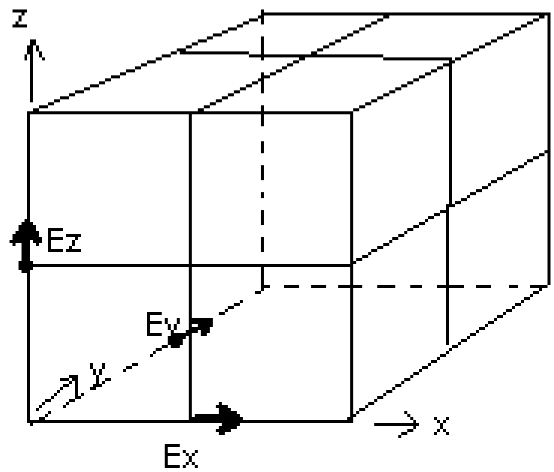
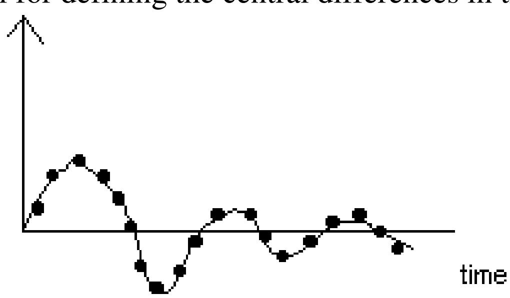
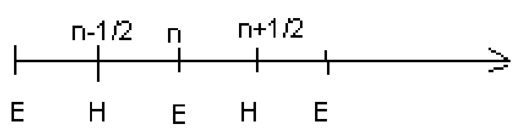


Discretize time (Δt):
Again, the E and H components will be at different times (Ex,Ey,Ez are defined at times $n\Delta t$, and Hx,Hy,Hz are defined at times $(n+1/2)\,\Delta t$). This is "leap frogging in time" and is useful for defining the central differences in time (d/dt)





Convert to derivative equations to difference equations using central difference formulas in space and time:
Be VERY specific about what LOCATION and TIME each equation is defined. This equation is defined at time $n\Delta t$ and the location of Hx on the Yee cell (see above).

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$$\frac{H_x^{n+1/2}(I,J,K)-H_x^{n-1/2}(I,J,K)}{\Delta t}=\frac{1}{\mu}\left[\frac{E_y^n(I,J,K+1)-E_y^n(I,J,K)}{\Delta z}-\frac{E_z^n(I,J+1,K)-E_z^n(I,J,K)}{\Delta y}\right]$$

The leapfrog schemes in space and time are critical to the central differences giving the derivatives at the space and time where the equation is defined.
Now, assume you know the field at the present and past times, find the field at the future time:

$$H_x^{n+1/2}(I,J,K)=H_x^{n-1/2}(I,J,K)+\frac{\Delta t}{\mu}\left[\frac{E_y^n(I,J,K+1)-E_y^n(I,J,K)}{\Delta z}-\frac{E_z^n(I,J+1,K)-E_z^n(I,J,K)}{\Delta y}\right]$$

This form of solving differential equations is referred to as an "initial value problem." Other types of solutions (like FDFD) may be referred to as "boundary value problems."

From our understanding of Maxwell's equations, we know that we MUST solve simultaneous for E and H (unless the solution is STATIC, which it must not be, since we are looking for changes in the time domain).

Convert E field equation to difference form.
This equation is a little trickier. It is defined at time (n+1/2)Δt and the location of Ex:

$$\frac{E_x^{n+1}(I,J,K)-E_x^n(I,J,K)}{\Delta t}=\frac{1}{\varepsilon}\left[\frac{H_z^{n+1/2}(I,J,K)-H_z^{n+1/2}(I,J-1,K)}{\Delta y}-\frac{H_{yz}^{n+1/2}(I,J,K)-H_y^{n+1/2}(I,J,K-1)}{\Delta z}\right]$$

$$-\frac{\sigma}{\varepsilon}\left[E_x^{n+1/2}(I,J,K)\right]$$

The trouble is, we don't have $Ex^{n+1/2}$
To get it, average E_x^n and E_x^{n+1}:

$$\frac{E_x^{n+1}(I,J,K)-E_x^n(I,J,K)}{\Delta t}=\frac{1}{\varepsilon}\left[\frac{H_z^{n+1/2}(I,J,K)-H_z^{n+1/2}(I,J-1,K)}{\Delta y}-\frac{H_{yz}^{n+1/2}(I,J,K)-H_y^{n+1/2}(I,J,K-1)}{\Delta z}\right]$$

$$-\frac{\sigma}{\varepsilon}\left[\frac{E_x^{n+1}(I,J,K)+E_x^n(I,J,K)}{2}\right]$$

Now, assume we know all fields at present and past times, solve for future time:

$$E_x^{n+1}(I,J,K)\left[1+\frac{\sigma\Delta t}{2\varepsilon}\right]=E_x^n(I,J,K)\left[1-\frac{\sigma\Delta t}{2\varepsilon}\right]+\frac{\Delta t}{\varepsilon}\left[\frac{H_z^{n+1/2}(I,J,K)-H_z^{n+1/2}(I,J-1,K)}{\Delta y}-\frac{H_{yz}^{n+1/2}(I,J,K)-H_y^{n+1/2}(I,J,K-}{\Delta z}\right.$$

$$E_x^{n+1}(I,J,K)=E_x^n(I,J,K)\left[1-\frac{\sigma\Delta t}{2\varepsilon}\right]/\left[1+\frac{\sigma\Delta t}{2\varepsilon}\right]+\frac{\Delta t}{\varepsilon\left[1+\frac{\sigma\Delta t}{2\varepsilon}\right]}\left[\frac{H_z^{n+1/2}(I,J,K)-H_z^{n+1/2}(I,J-1,K)}{\Delta y}-\frac{H_{yz}^{n+1/2}(I,J,K)-H_y^{n+1/2}}{\Delta z}\right.$$

This might seem like we are magically PREDICTING the future from the past. This is EXACTLY what we are doing. Maxwell's equations are physical laws that describe the

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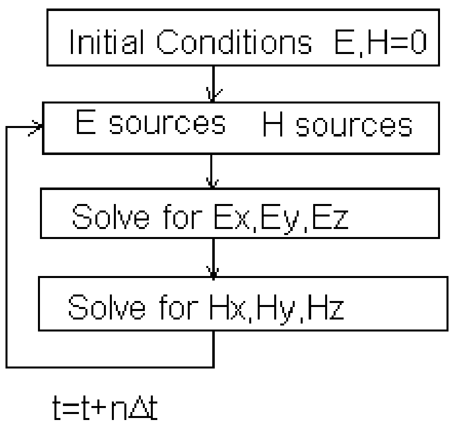
motion of waves. E and H waves obey these laws, so we can use them to PREDICT what E and H fields will be generated by the source fields.

FDTD Algorithm:



What do you program?
Store arrays of Ex,Ey,Ez, Hx,Hy,Hz at every location in the model. Also store physical parameters ε,μ,σ at every location in the model.
OMIT indices n, n+1/2, n+1, etc. by overwriting arrays as they are calculated. (Note that you never need the old value of the fields after the new value has been calculated.)

About iterations:
Make a big FOR loop that iterates the fields as a function of time (n). These iterations are totally different than the iterations from SOR/FDFD. In SOR/FDFD we were iterating to improve the guess of the solution to a matrix equation. The intermediate iterations gave field values that were NOT CORRECT.
In FDTD, the iterations are as a function of time, and each iteration computes field values that are CORRECT (to order $(\Delta x)^2$) for each iteration.

What you see: frame-by-frame "movie" of the fields being scattered by an object.

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