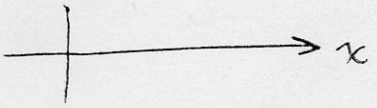
1D-TE FDTD EQUATIONS



$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0 \qquad E_z = 0$$

TE

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\cancelto{0}{\frac{\partial H_z}{\partial y}} - \cancelto{0}{\frac{\partial H_y}{\partial z}} - \sigma E_x\right) \quad (1) \qquad \frac{\partial E_x}{\partial t} = \frac{-\sigma E_x}{\varepsilon}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\cancelto{0}{\frac{\partial H_x}{\partial z}} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \quad (2) \qquad \frac{\partial E_y}{\partial t} = \frac{-1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + \sigma E_y\right.$$

$$\frac{\partial \cancelto{0}{E_z}}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \cancelto{0}{\frac{\partial H_x}{\partial y}} - \sigma \cancelto{0}{E_z}\right) \quad (3) \qquad 0 = \frac{\partial H_y}{\partial x}$$

$$\frac{\partial H_x}{\partial t} = \frac{-1}{\mu}\left(\cancelto{0}{\frac{\partial E_z}{\partial y}} - \cancelto{0}{\frac{\partial E_y}{\partial z}}\right) \quad (4) \qquad \frac{\partial H_x}{\partial t} = 0$$

$$\frac{\partial H_y}{\partial t} = \frac{-1}{\mu}\left(\cancelto{0}{\frac{\partial E_x}{\partial z}} - \cancelto{0}{\frac{\partial E_z}{\partial x}}\right) \quad (5) \qquad 0 = \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_z}{\partial t} = \frac{-1}{\mu}\left(\frac{\partial E_y}{\partial x} - \cancelto{0}{\frac{\partial E_x}{\partial y}}\right) \quad (6) \qquad \frac{\partial H_z}{\partial t} = \frac{-1}{\mu}\frac{\partial E_y}{\partial x}$$

From eqn (5)

$\frac{\partial H_y}{\partial t} = 0$ and H_y is initially 0, so $H_y = 0$ always

From eqn (4)

Similar to above $H_x = 0$
This also eliminates eqn (3)

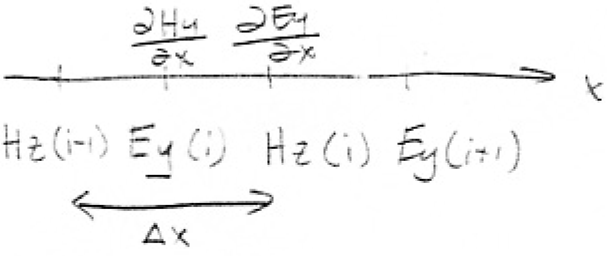
From (1)

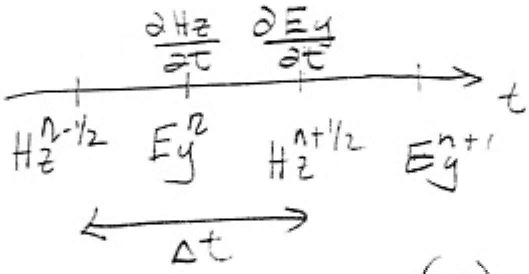
This 1D code models a plane wave source, only.
For an x-propagating plane wave, there are no components in the x-direction. $E_x = 0$

1D TE differential equations

$$(7)\quad \frac{\partial E_y}{\partial t} = \frac{-1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + \sigma E_y\right)$$

$$(8)\quad \frac{\partial H_z}{\partial t} = \frac{-1}{\mu}\,\frac{\partial E_y}{\partial x}$$





Apply central difference to (7):

$$\frac{E_y^{n+1} - E_y^{n}}{\Delta t} = \frac{-1}{\varepsilon_i}\left(\frac{H_z^{n+1/2}(i) - H_z^{n+1/2}(i-1)}{\Delta x} + \sigma\,\frac{E_y^{n+1}(i) + E_y^{n}(i)}{2}\right)$$

Apply central difference to (8):

$$\frac{H_z^{n+1/2}(i) - H_z^{n-1/2}(i)}{\Delta t} = \frac{-1}{\mu_i}\left(\frac{E_y^{n}(i+1) - E_y^{n}(i)}{\Delta x}\right)$$

Solve for:

$$E_y^{n+1}(i) = \underbrace{\left(\frac{1 - \frac{\Delta t \sigma_i}{2\varepsilon_i}}{1 + \frac{\Delta t \sigma_i}{2\varepsilon_i}} \right)}_{C1(i)} E_y^{n}(i)$$

$$+ \underbrace{\left(\frac{\frac{\Delta t}{\Delta x \, \varepsilon_i}}{1 + \frac{\Delta t \sigma}{2\varepsilon_i}} \right)}_{C2(i)} \left(H_z(i-1) - H_z(i) \right)$$

$$H_z^{n+1/2}(i) = H_z^{n-1/2}(i) - \underbrace{\frac{\Delta t}{\mu_i \Delta x}}_{d1(i)} \left(E_y^n(i+1) - E_y^n(i) \right)$$

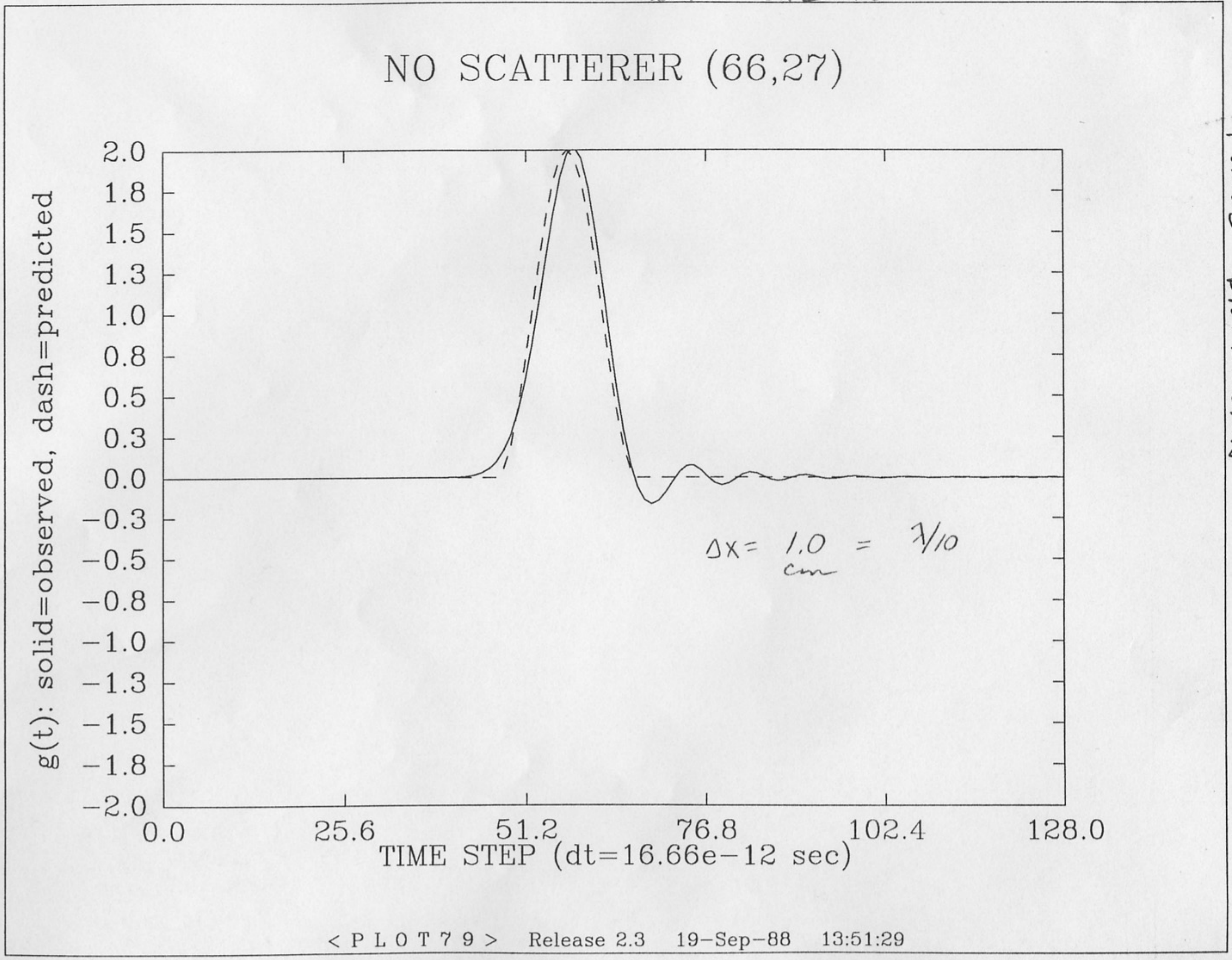
* C1, C2, D1 depend on the $\varepsilon_i, \mu_i, \sigma_i$ values in each cell, and are therefore arrays.

* For many FDTD codes, the parameter $\Delta t = \frac{\Delta x}{2 c_0}$ is "hard-coded" into these constants, so they do not appear to depend on Δt, when in fact they do.

* For many FDTD codes, H fields are multiplied $\eta = 377$. to increase numerical accuracy

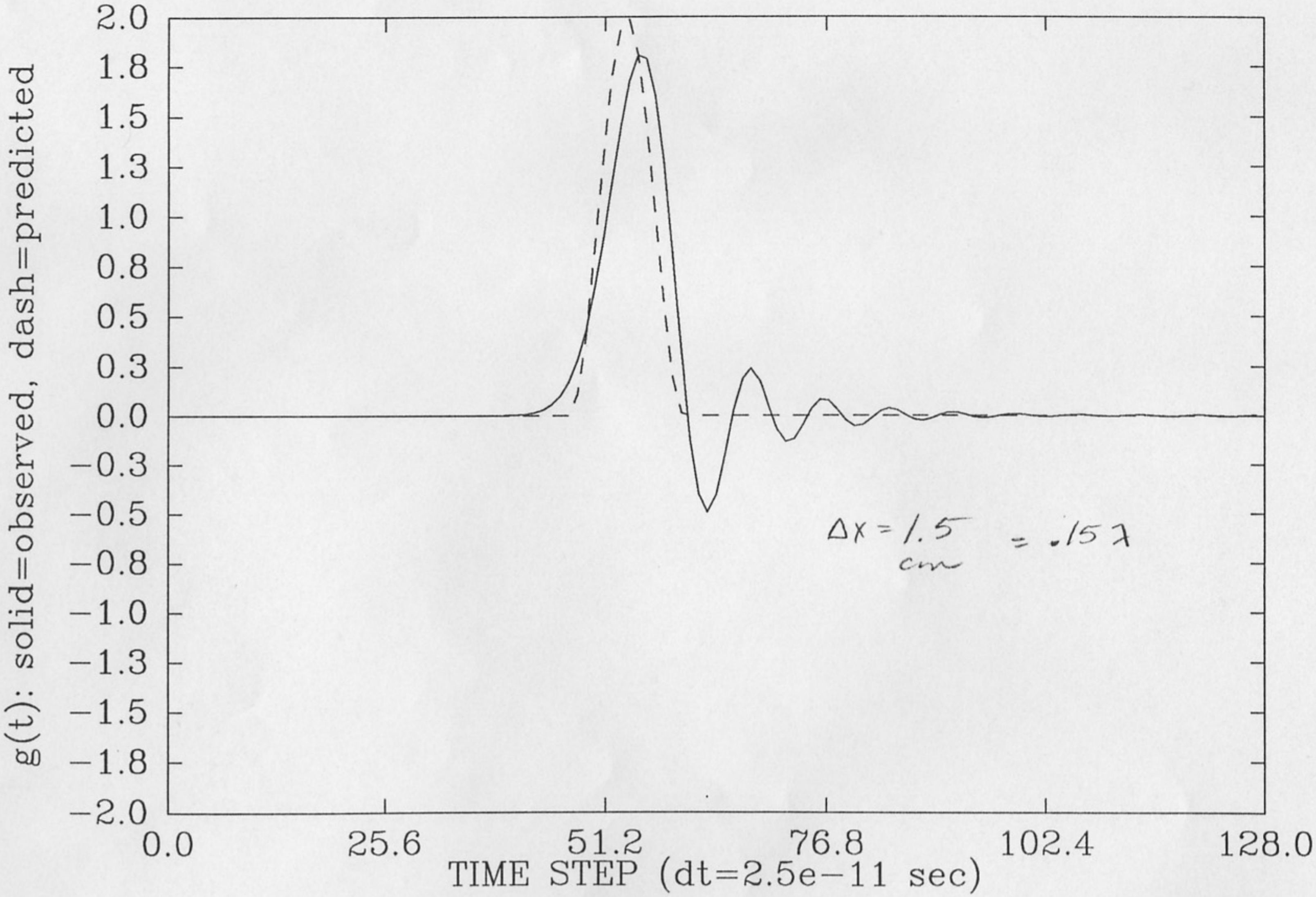
PULSE DISPERSION

Demonstrate Pulse Dispersion
when $\Delta x \leq \lambda/10$



5

NO SCATTERER (66,27)



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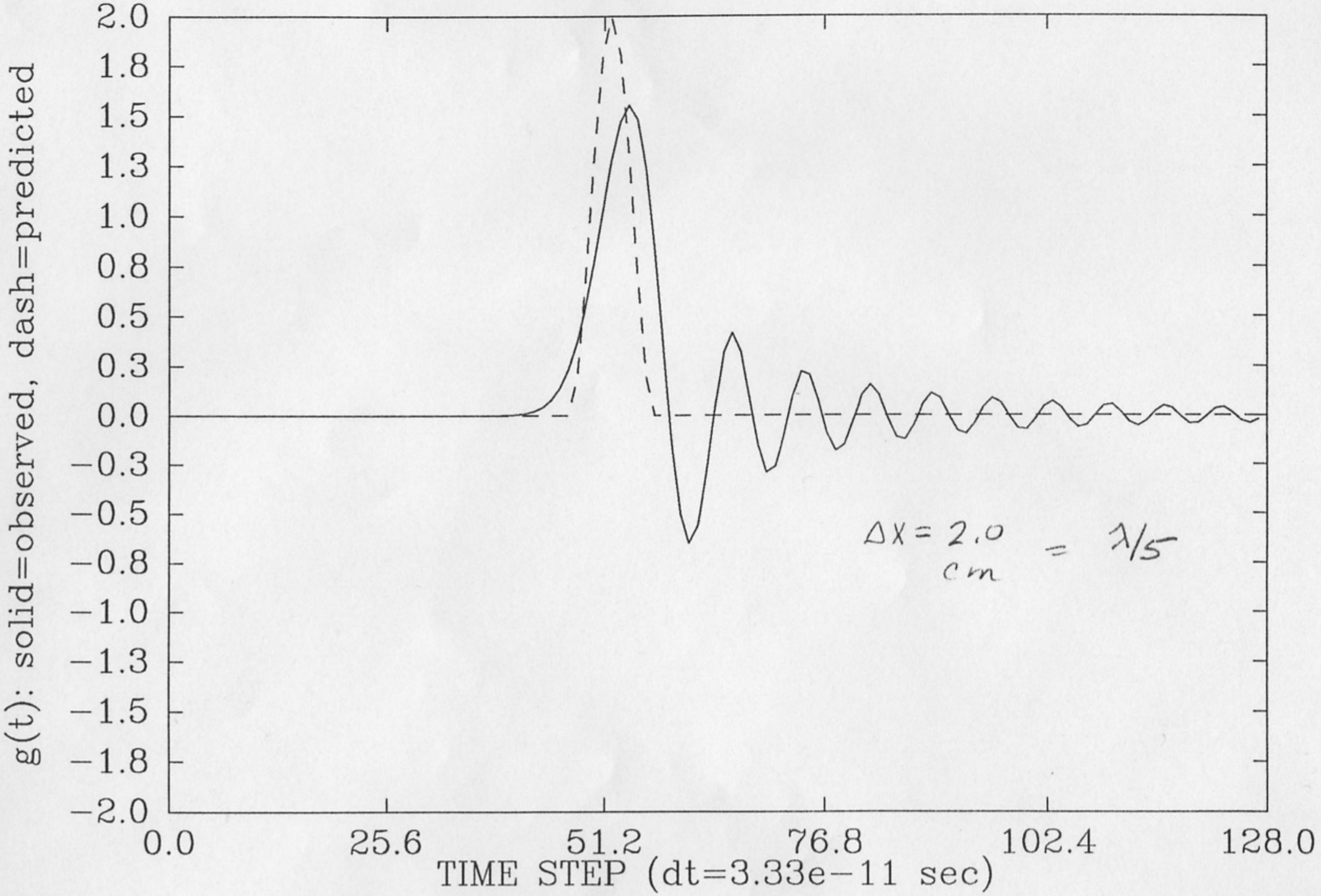
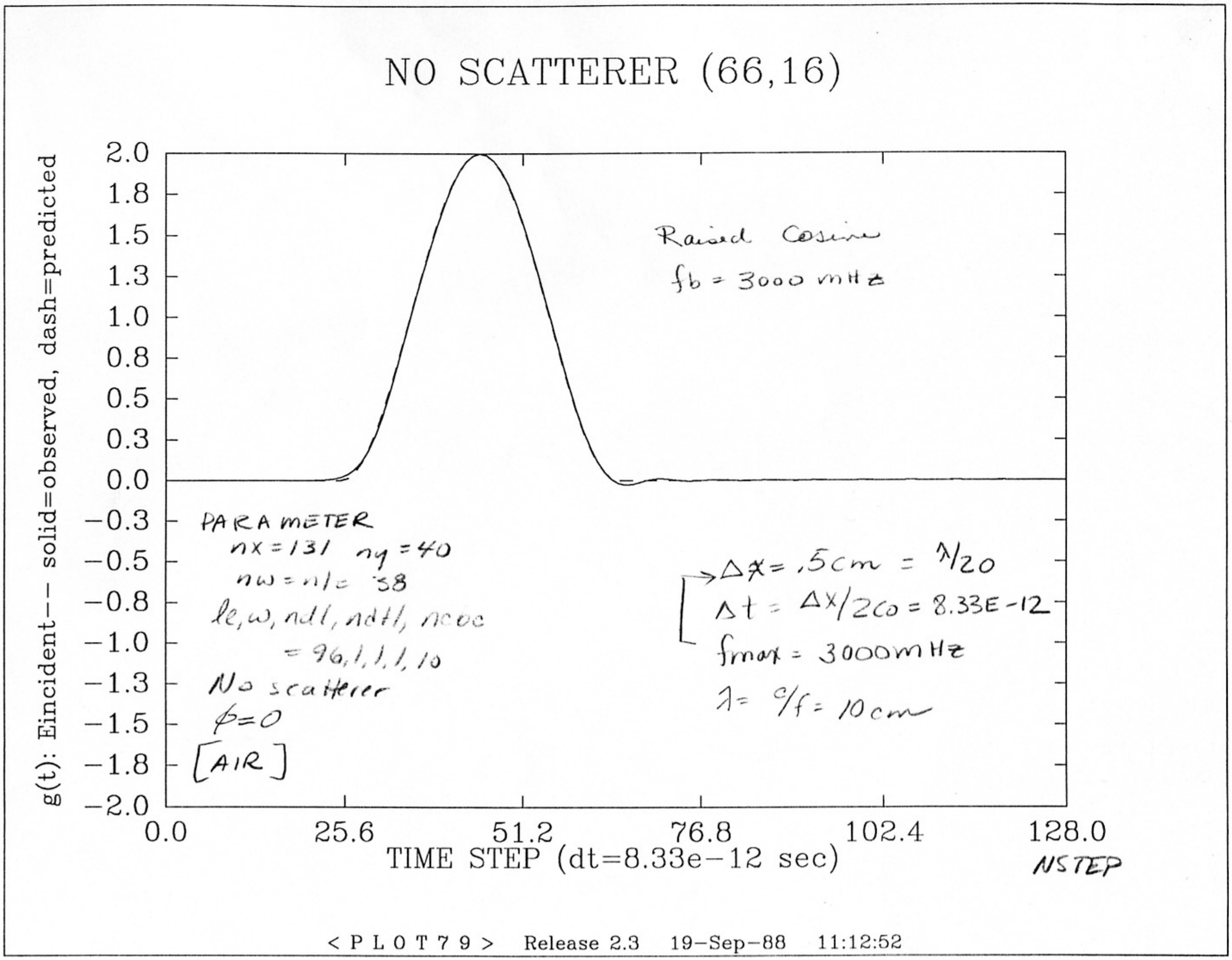
NO SCATTERER (66,27)




NO SCATTERER (66,16)
g(t): Eincident-- solid=observed, dash=predicted
2.0
1.8
1.5
1.3
1.0
0.8
0.5
0.3
0.0
−0.3
−0.5
−0.8
−1.0
−1.3
−1.5
−1.8
−2.0
0.0
25.6
51.2
76.8
102.4
128.0
TIME STEP (dt=8.33e−12 sec)
NSTEP
Raised Cosine
fb = 3000 mHz
PARAMETER
nx = 131 ny = 40
nw = n/c 38
le, w, ndl, ndtl, ncoc
= 96, 1, 1, 1, 10
No scatterer
φ = 0
[AIR]
Δx = .5cm = λ/20
Δt = Δx/2co = 8.33E-12
fmax = 3000mHz
λ = c/f = 10cm
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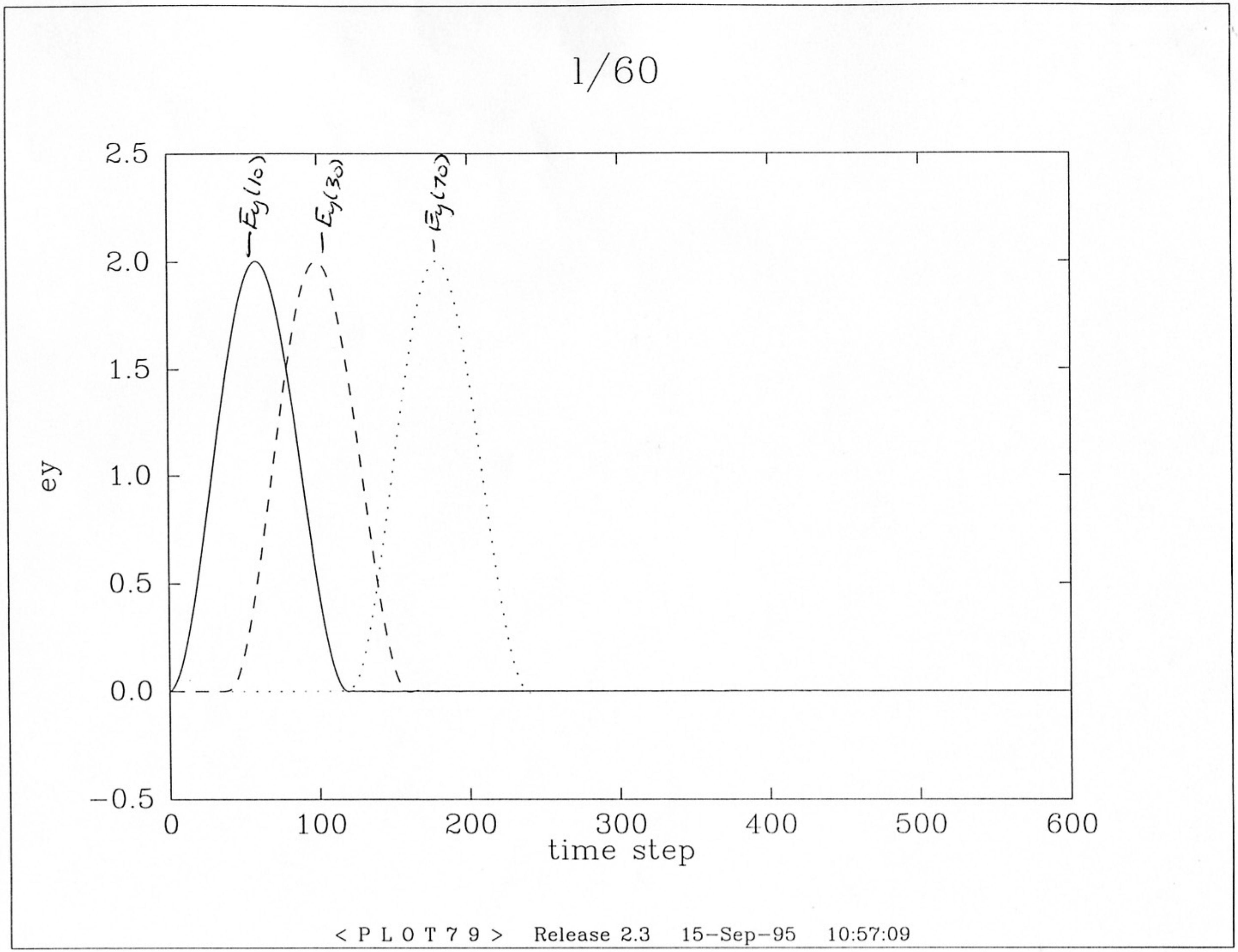
1/60
$E_y(10)$
$E_y(30)$
$E_y(70)$
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
ey
0
100
200
300
400
500
600
time step
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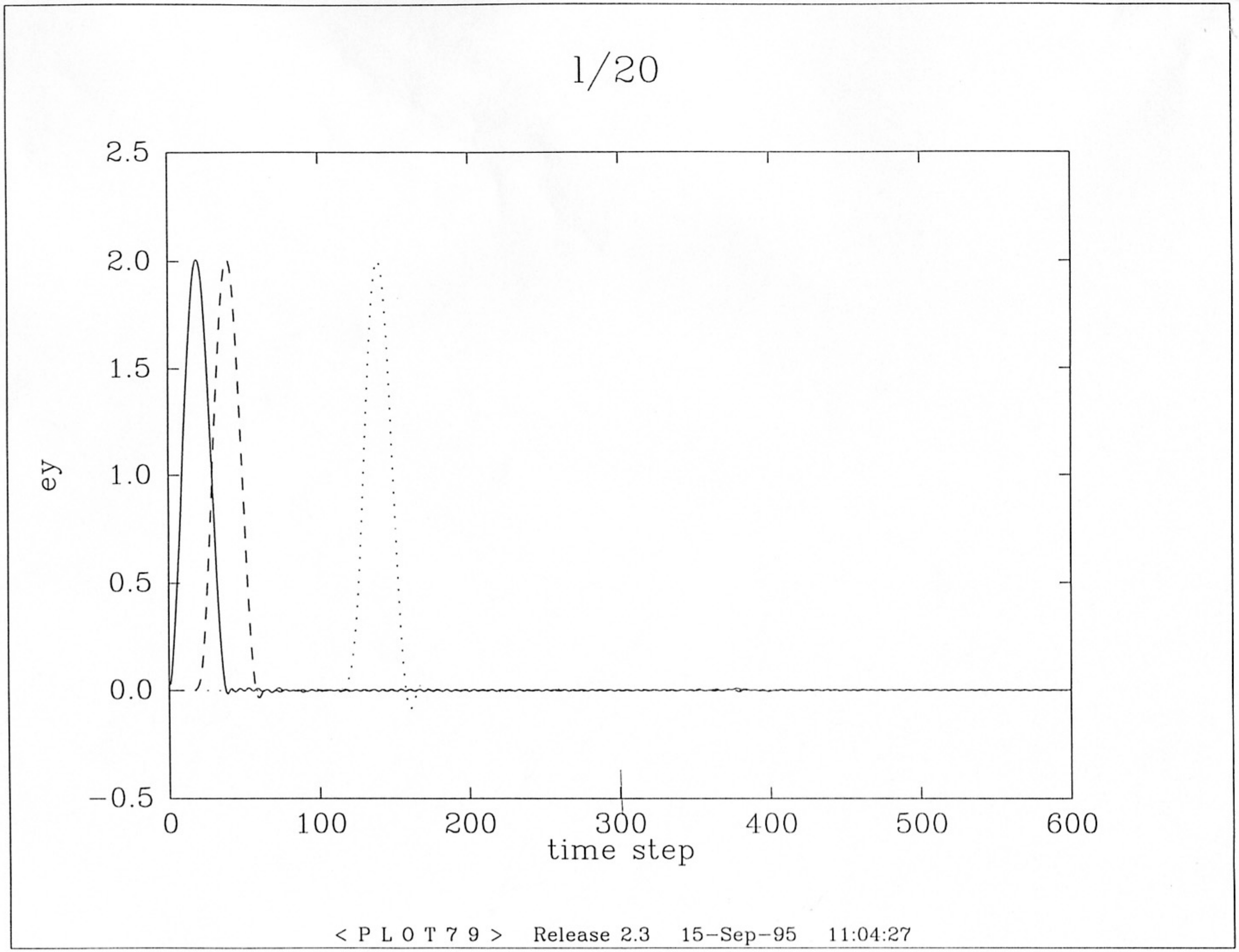
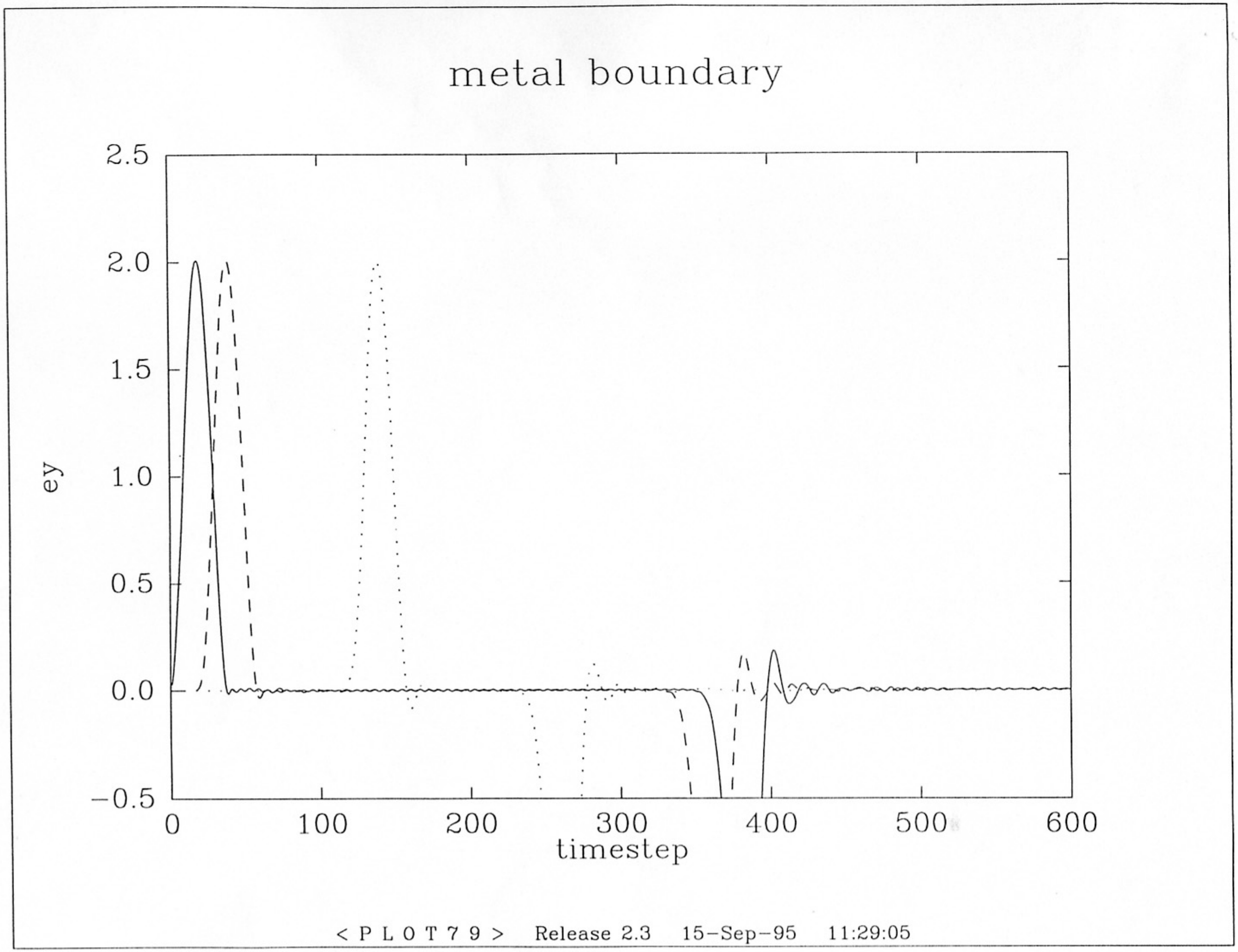
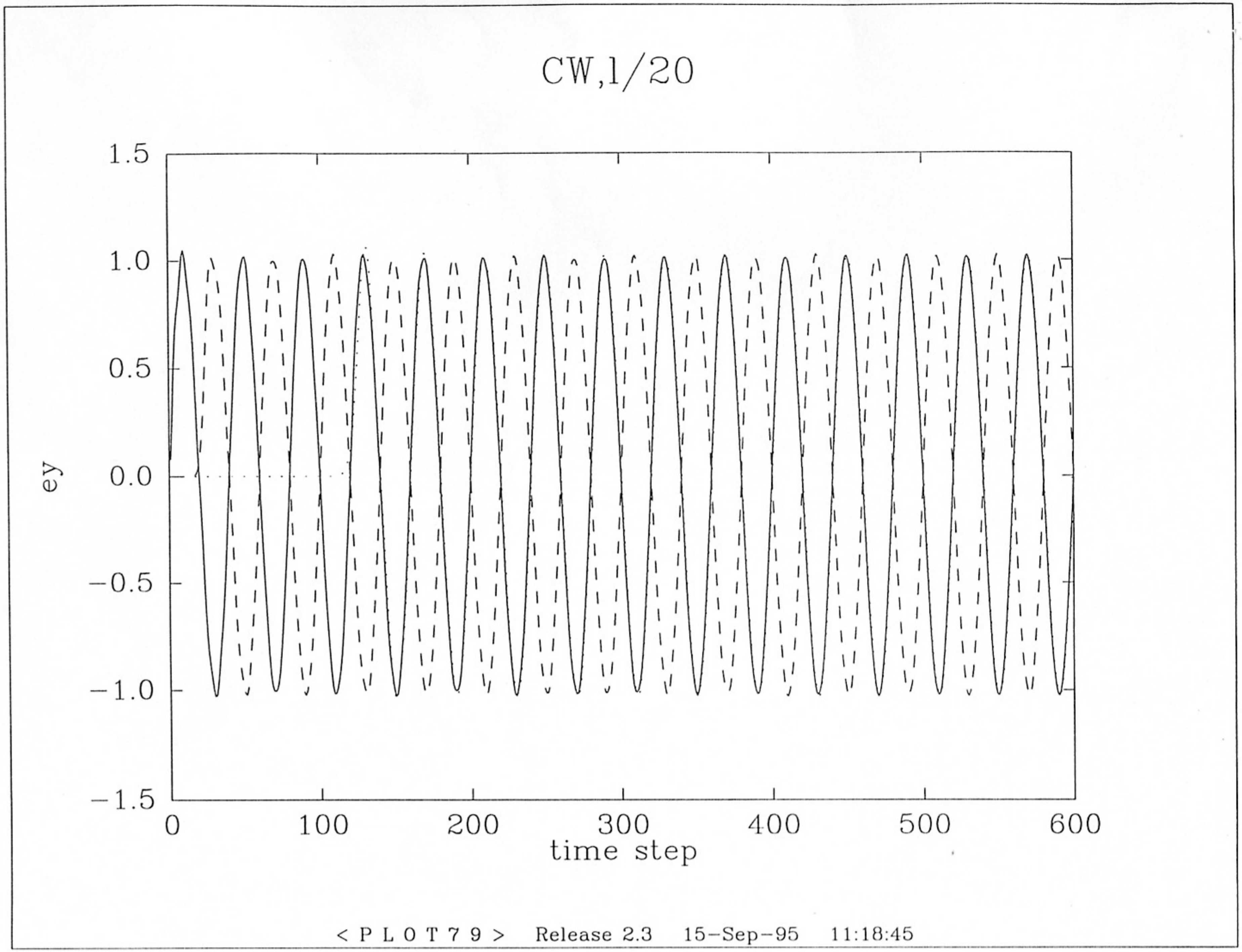

1/20
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
ey
0
100
200
300
400
500
600
time step
< P L O T 7 9 > Release 2.3 15−Sep−95 11:04:27

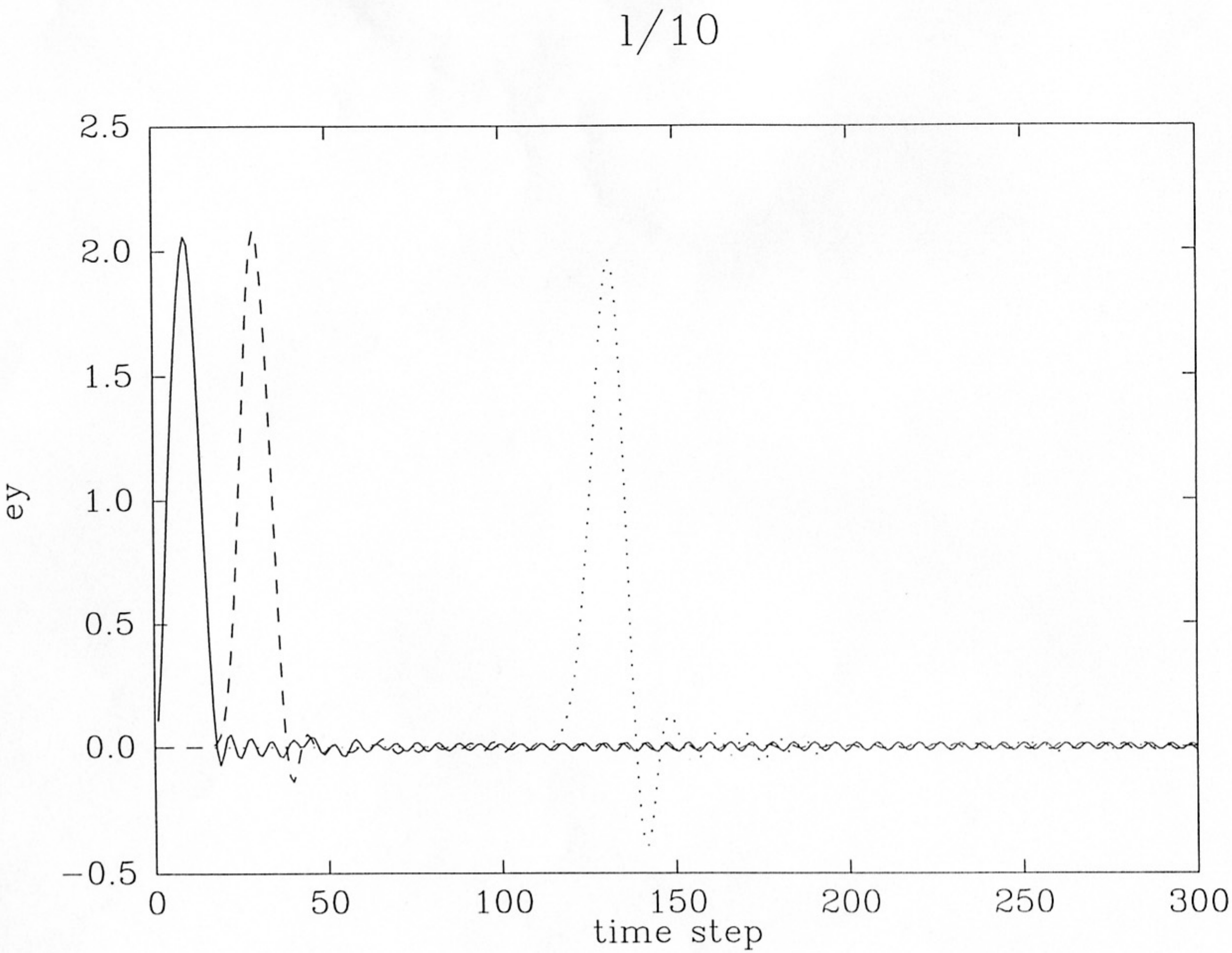
(b)


metal boundary
ey
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0
100
200
300
400
500
600
timestep
< P L O T 7 9 >   Release 2.3   15−Sep−95   11:29:05

(10)


CW,1/20
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
ey
0
100
200
300
400
500
600
time step
< P L O T 7 9 > Release 2.3 15−Sep−95 11:18:45

(11)



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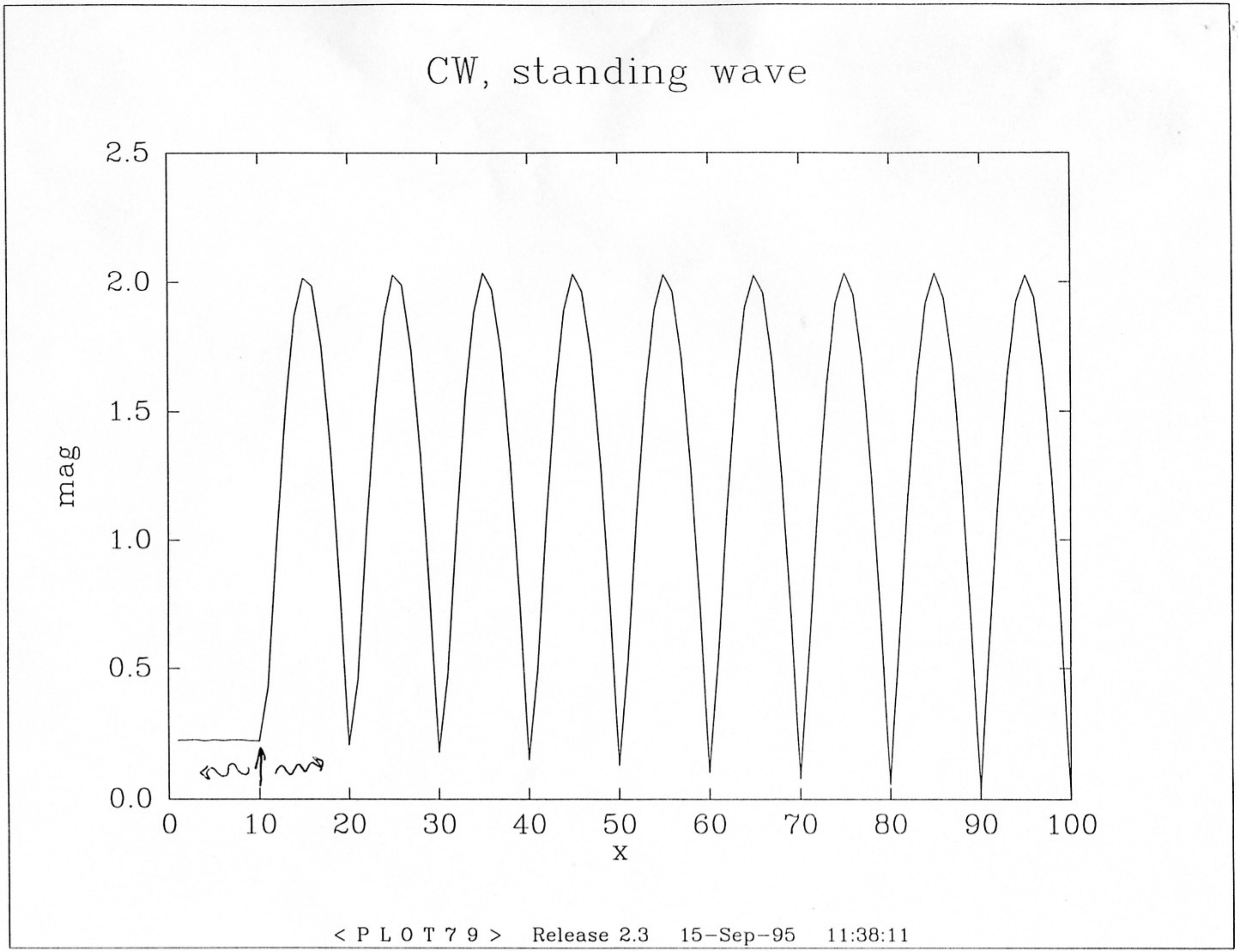
CW, standing wave
mag
2.5
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
50
60
70
80
90
100
X
< P L O T 7 9 >   Release 2.3   15-Sep-95   11:38:11