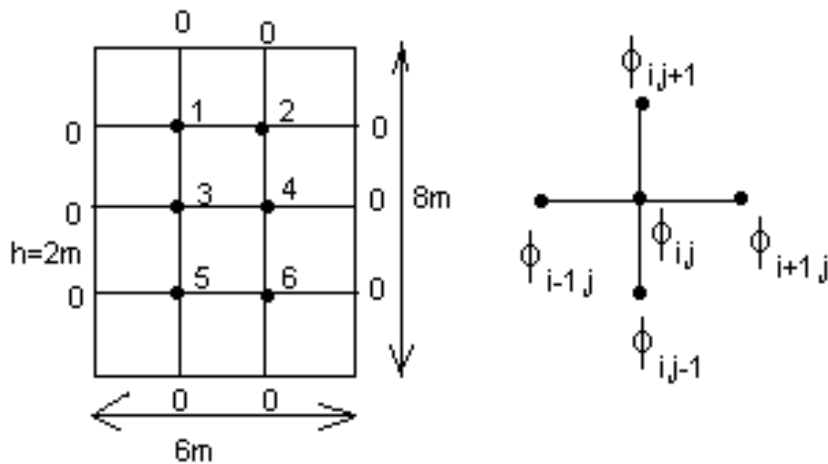


ECE 5340/6340 FDFD: Poisson's Equation and Use of SOR

EXAMPLE: Use of FDFD for Solving Poisson's Equation

Poisson's Equation:

$$\nabla^2 \phi = -q_v / \epsilon = -2$$



Replace Laplacian $\nabla^2 \phi$ with its difference equation:

$$\nabla^2 \phi = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1} - 4\phi_{i,j}}{h^2}$$

Node 1: $[0+0+\phi_3+\phi_2-4\phi_1]/h^2 = -2 \ll$ Poisson's

Node 2: $[0+\phi_1+\phi_4+0-4\phi_2]/h^2 = -2$

Node 3: $[\phi_1+0+\phi_5+\phi_4-4\phi_3]/h^2 = -2$

Bring h^2 to RHS

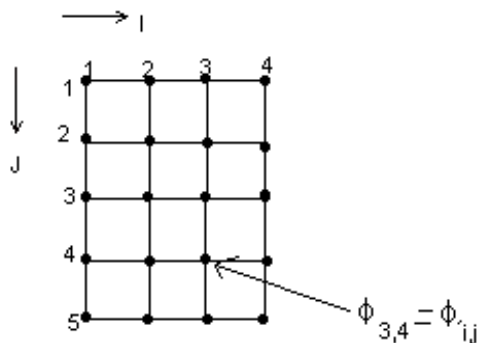
$$\begin{bmatrix} -4 & 1 & 1 & 0 & 0 & 0 \\ 1 & -4 & 0 & 1 & 0 & 0 \\ 1 & 0 & -4 & 1 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 & 1 \\ 0 & 0 & 1 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{bmatrix} = \begin{bmatrix} -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \end{bmatrix}$$

$$\phi_1 = \phi_2 = \phi_5 = \phi_6 = 4.57143 \text{ V}; \phi_3 = \phi_4 = 5.71429 \text{ V}$$

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SOLVE USING SOR:

Redefine grid numbering for ease of programming



1) Set Boundary Conditions:

$$\phi_{11} = \phi_{12} = \phi_{13} = \phi_{14} = \phi_{15} = 0 \text{ (Left)}$$

$$\phi_{41} = \phi_{42} = \phi_{43} = \phi_{44} = \phi_{45} = 0 \text{ (Right)}$$

$$\phi_{21} = \phi_{31} = 0 \text{ (Top)}$$

$$\phi_{25} = \phi_{35} = 0 \text{ (Bottom)}$$

2) Re-write FDFD algorithm in terms of SOR form and apply SOR Algorithm to all interior points:

$$\phi_{i,j}^{(k+1)} = \phi_{i,j}^{(k)} + \omega R_{i,j}^{(k)}$$

$$R_{i,j}^{(k)} = \frac{\phi_{i+1,j}^{(k)} + \phi_{i-1,j}^{(k)} + \phi_{i,j+1}^{(k)} + \phi_{i,j-1}^{(k)} - 4\phi_{i,j}^{(k)} - b_{i,j}}{4}$$

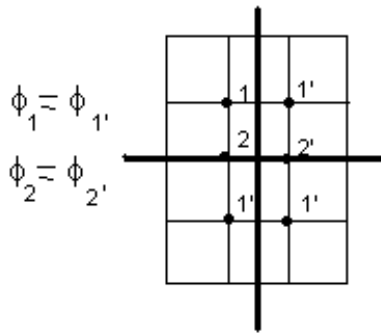
$$b_{i,j} = -h^2 q_v / \varepsilon$$

Handouts:

Repeat of FDFD Example using SOR

Fortran code to solve FDFD using SOR

SYMMETRY:



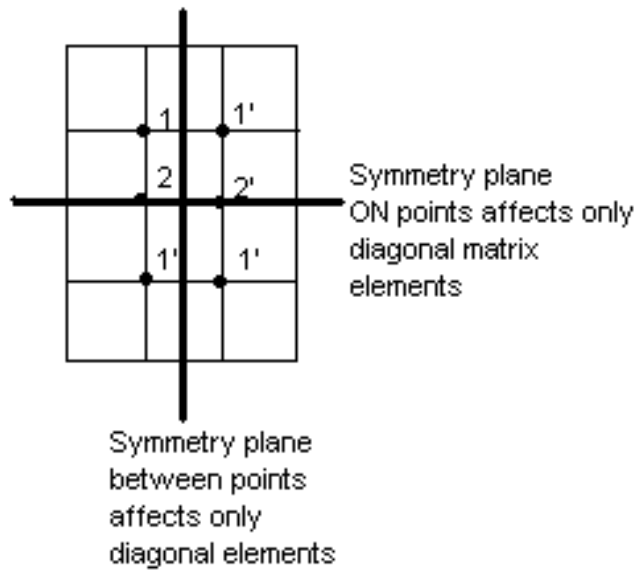
$$\text{Node 1: } h^2[0 + \phi_1' + \phi_2 + 0 - 4\phi_1] = -2 \quad ; h=2$$

$$-3\phi_1 + \phi_3 = -8$$

$$\text{Node 2: } h^2[\phi_1 + \phi_3' + \phi_1' + 0 - 4\phi_3] = -8$$

$$2\phi_1 - 3\phi_2 = -8$$

Observe:



EXAMPLE: See programming handout using symmetry