S(

cm. A short circuit is now substituted for the load, and minima are found at 7 cm, 19 cm, and 31 cm. Find the frequency being transmitted and the load impedance, using the Smith chart. Assume that $Z_0 = 50\ \Omega$.

Solution. The frequency is calculated from the distance between minima. Knowing it is an air-dielectric line, 12 (cm) is $\lambda/2$ in air, and

$$f = \frac{3 \times 10^8 \text{ m s}^{-1}}{2(0.12) \text{ (m)}} = \frac{U}{\lambda}$$
$$= 1.25 \times 10^9 = 1.25 \text{ GHz}$$

A location of a minima with the short substituted for a load can be used as a load position reference. Any one of these minima may be used for such a reference, since $Z_L = 0$ is repeated every $\lambda/2$ from the short circuit, and the position of the short circuit is the position of the load when it is connected to the line. Then, referring to Fig. 12-26(a), with the load connected a minimum is found 10 cm toward the generator from the load on the $S = 2.3$ circle. Since the wavelength is 24 cm, 10 cm is 0.42λ. Then, measuring 0.42λ from V_{min}, on the impedance Smith chart we read that

$$z_L = 0.54 + j0.425$$

or

$$Z_L = Z_0 z_L = 50 z_L = 27 + j21.25 \qquad (\Omega)$$

as illustrated in Fig. 12-26(b).

Problem 12-14-1 With a load connected directly to the end of a 50-Ω slotted line, the measured S is 2.0. When a short is substituted for the load, the position of the minima is shifted 1 (cm) toward the load, and the distance between minima is 8 (cm). Find the frequency and the load impedance.

Problem 12.14-2 The same data is obtained as in Prob. 12.14-1, except that the position of the minima is shifted 1 (cm) toward the generator when the short is substituted for the load. Find the frequency and the load impedance.

12.15 LOSSY LINE ANALYSIS USING THE SMITH CHART

The load reflection coefficient was given by (12.4-13) as

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad (12.4\text{-}13)$$

or

$$\rho = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1} \qquad (1)$$

in normalized form. A generalized reflection coefficient may be defined at the input to the transmission line as

$$\rho_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \qquad (2)$$

where Z_{in} is defined by (12.4-18). However, it is more convenient for the present purpose to use Z_{in} in the form

$$Z_{in} = \frac{Z_0(1 + \rho e^{-2\gamma\ell})}{1 - \rho e^{-2\gamma\ell}} \qquad (3)$$

12.15
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which when substituted into (2), and with some algebraic simplification, reduces (2) to

$$\rho_{in} = \rho e^{-2\gamma\ell} \tag{4}$$

or

$$\rho_{in} = (|\rho|\angle\theta_{\rho L})(e^{-2\alpha\ell}\angle -2\beta\ell) \tag{5}$$

for the reflection coefficient at a distance ℓ from the load. The input impedance at the given distance from the load is treated as a load impedance at that location. Keep in mind that on the Smith chart, the radius from the center of the chart to the coordinates of the normalized impedance under consideration is $|\rho_{in}|$ for that impedance, where the measure of $|\rho_{in}|$ is scaled from the voltage reflection coefficient scale at the bottom of the Smith chart. The angle of the complex ρ_{in} is read directly from the angular position of that radius. One can easily construct a loci of z_{in} for various ℓ using (5), provided that ρ, α, and β are known. The lossy transmission line of Example 5, with a very mismatched load of $Z_L = 50 + j50\ \Omega$, will be used to illustrate.

Example 18

For a frequency of 1000 Hz as in Example 5, the numerical values for Z_0, α, and β were found to be

$$Z_0 = 612\angle -5.35^\circ\ \Omega = 609.85 - j57.09\ \Omega$$
$$\alpha = 0.00345 \text{ Np/mile} = 0.02998 \text{ dB mile}^{-1}$$
$$\beta = 0.03504 \text{ rad/mile} = 2.008^\circ \text{ mile}^{-1}$$

We want to calculate and plot on a Smith chart values for ρ_{in} for $0 < \ell < 200$ miles.

Solution. First, the load reflection coefficient is found to be

$$\rho = \frac{50 + j50 - 609.85 + j57.09}{50 + j50 + 609.85 - j57.09} = 0.864\angle 169.787^\circ$$

At 5 miles,

$$\rho_{in} = (0.864\angle 169.787^\circ)[e^{-(2)(5)(0.00345)}\angle -2(5)(2.008)] = 0.835\angle 149.707^\circ$$

At 30 miles,

$$\rho_{in} = (0.864\angle 169.787^\circ)[e^{-(2)(30)(0.00345)}\angle -2(30)(2.0)] = 0.702\angle 49.307^\circ$$

In a similar manner, ρ_{in} is calculated and plotted for a number of distances ℓ in Fig. 12-27, creating a spiral that approaches the center of the chart as ℓ approaches ∞.

To find Z_{in}, read z_{in} directly from the chart and multiply by Z_0. For example, at 20 miles, ρ_{in} was found to be $0.753\angle 89.467^\circ$. When plotted on the Smith chart of Fig. 12-27, we read

$$z_{in} = 0.28 + j0.97 = 1.01\angle 73.89^\circ$$
$$Z_{in} = (612\angle -5.35^\circ)1.01\angle 73.89^\circ = 617.88\angle 68.55^\circ \quad (\Omega)$$

compared to $616\angle 68.1^\circ\ \Omega$ calculated in Example 5. Of course, some accuracy must be sacrificed when using the Smith chart for such calculations.

The transmission loss (1-dB steps) scale may be used to measure the transmission loss between various points on the spiral. For example, find the loss between 90 and

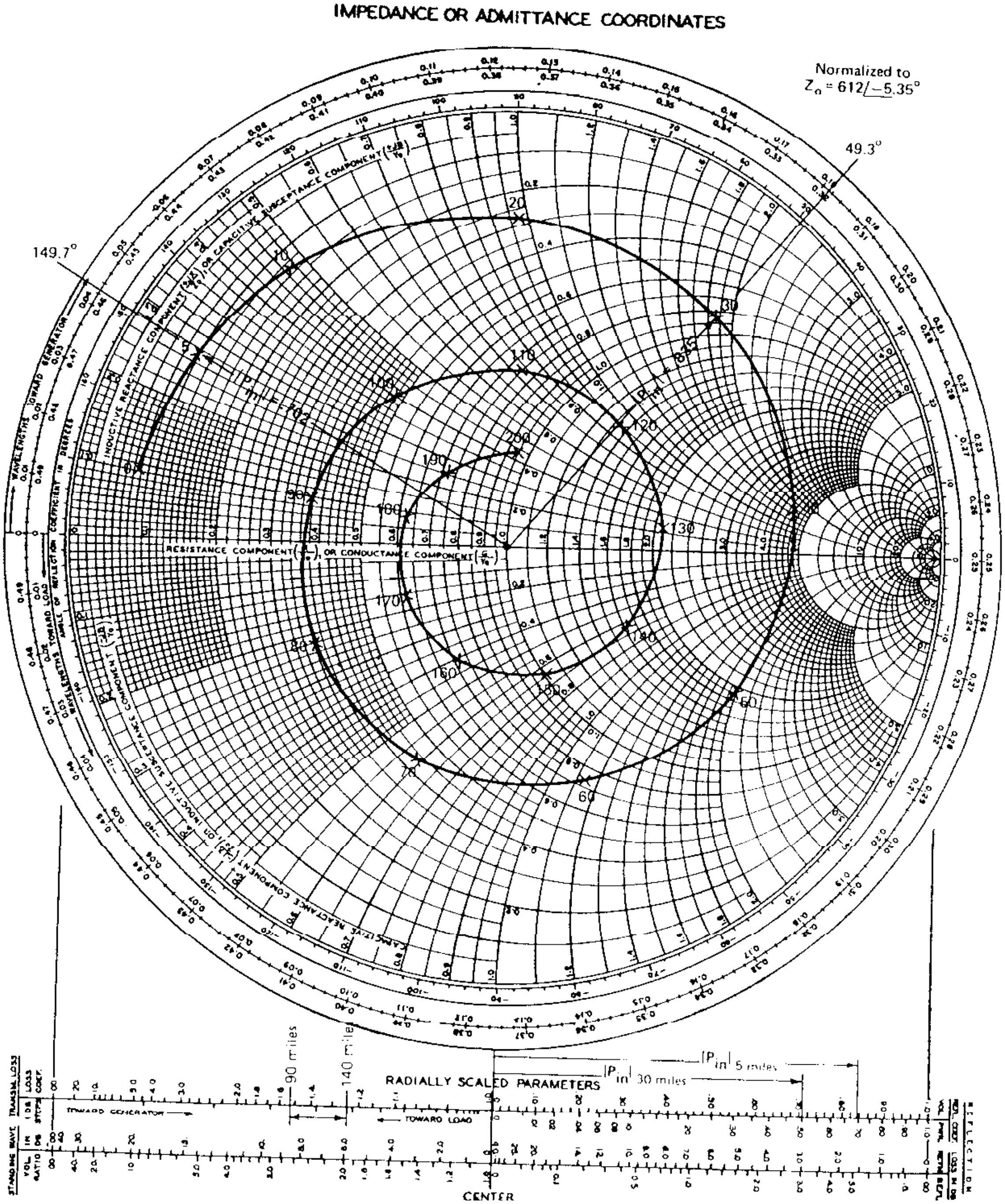

Figure 12-27 Illustration for Example 18.

140 miles. Using the transmission loss scale to measure the radius of the spiral, we read approximately 1.5 dB for the difference in radii. To check this value, we calculate

$$10 \log_{10} e^{-2\alpha l} = 10 \log_{10} e^{-2(0.00345)(50)} = -1.5 \text{ dB}$$

which is independent of which 50-mile section is considered.

Each time the spiral of Fig. 12-27 completes 360°, the attenuation corresponds to that for one-half wavelength of transmission line. For this particular line of Example 18, $\lambda/2 = 180°/(2.008° \text{ mile}^{-1}) = 90$ miles. Thus, for every 90 miles of travel, or for each revolution of the plot, the radius of the plot, i.e., the magnitude of the reflection coefficient, will be reduced by a factor $e^{-90(0.00345)} = 0.73$ and the attenuation in dB is $-10 \log_{10}(0.73)^2 = 2.7$ dB.

The standing wave ratio may be determined for any point on the spiral by drawing a standing wave circle through that point as was done for a lossless line. It is obvious that the reflection coefficient and standing wave ratio decrease with increasing distance from the load.

Problem 12.15-1 Using the spiral plot of Fig. 12-27 and Example 18, estimate the shortest distance from the load to where Z_{in} is purely real, and determine that value of Z_{in}. Check your estimate by calculating Z_{in} using (12.4-18).

Problem 12.15-2 Determine the standing wave ratio for Example 18 at a distance of 180 miles from the load.

Problem 12.15-3 Find Z_{in} for Example 18 as $\ell \to \infty$.

REVIEW QUESTIONS

1. What is the TEM mode? *Sec. 12.1*
2. What is meant by distributed parameters as contrasted to lumped parameters? *Secs. 12.1, 12.2*
3. What is characteristic impedance? Propagation constant? *Secs. 12.2, 12.4*
4. How does one calculate hyperbolic functions of complex numbers? *Example 5*
5. Qualitatively, what distinguishes a power transmission line from a communications-type line? *Sec. 12.5*
6. What is a distortionless line? If R and G are both zero, is the line distortionless? *Sec. 12.6*
7. How does the velocity of propagation along a lossless line compare to that of a plane wave? *Sec. 12.8*
8. How does the characteristic impedance of a lossless line relate to the intrinsic impedance of a medium? *Sec. 12.8*
9. How are S, V_{max}, V_{min}, $(Z_{in})_{max}$, $(Z_{in})_{min}$, and $|\rho|$ related for a lossless line? *Sec. 12.9*
10. For an adjustable-length lossless line with a load Z_L, how many distinct values of Z_{in} are there, in general, where Z_{in} is entirely real? *Sec. 12.9*
11. What is the range of possible input impedance values for a shorted line? *Sec. 12.9*
12. What is a quarter-wave transformer? Is it broad band? *Sec. 12.10*
13. How does the maximum power handling capability vary with the standing wave ratio? *Sec. 12.11*
14. Why are the coordinates on the Smith chart normalized?
15. Name some advantages, if any, of the Smith chart over a rectangular chart. *Sec. 12.12*
16. For a transmission line connected to a mismatched load, how many locations, within a $\lambda/2$ interval, could a single shunt stub be used to achieve a match between the source and the stub? *Sec. 12.13*
17. What advantages does the double-stub tuner have over the single-stub tuner? *Sec. 12.13*
18. Can one always achieve a match with an arbitrary location of the double stub tuner? *Sec. 12.13*
19. Describe the basic procedure for using a slotted line to measure load impedance. *Sec. 12.14*
20. How does skin effect affect the loss of a transmission line? *Sec. 12.7*
21. When a step change of voltage is applied to a transmission line, what determines the instantaneous change in input current? *Sec. 12.3*