ECE 6130 -- TEM / TE / TM Fields

Text Sections 3.1

Portfolio Question. What are TEM, TE, and TM waves?
Do Chapter 3, Problem 1

TEM (Transverse Electro-magnetic) Waves

In transmission lines, we considered TEM propagation. This meant E and H were both perpendicular (transverse) to direction of propagation. Now we will generalize this, and consider TE and TM modes also. Fields in a waveguide (closed cavity) or transmission line are generalized to the SUM of combinations of TEM, TE, TM modes. At the lowest resonant frequency, only one mode will resonate. As the frequencies become higher, the lowest modes and additional higher-order modes MAY be excited. Which modes will actually be excited depends on the source location and type.

General Wave propagating in lossless material (α=0) the +z direction:

$\mathbf{E} = (Ex\ \mathbf{x} + Ey\ \mathbf{y} + Ez\ \mathbf{z})\ e^{-j\beta z} = (Et\ \mathbf{t} + Ez\ \mathbf{z})\ e^{-j\beta z}$

(transverse direction = x and / or y)

$\mathbf{H} = (Hx\ \mathbf{x} + Hy\ \mathbf{y} + Hz\ \mathbf{z})\ e^{-j\beta z} = (Ht\ \mathbf{t} + Hz\ \mathbf{z})\ e^{-j\beta z}$

Maxwell's Equations in source-free region (no external charges or currents):

$\nabla \times \mathbf{E} = -j\omega\mu\ \mathbf{H}$

$\nabla \times \mathbf{H} = j\omega\varepsilon\ \mathbf{E}$

For z-dependence: $e^{-j\beta z}$, $d/dz = -j\beta$

Apply to Maxwell's Equations and equate vector components.

VIEWGRAPH and handouts of equations 3.3-3.4

We can solve (homework problem) for the transverse (x and y) components as functions of the z components:
VIEWGRAPH of equations 3.5 -- GENERAL field solutions

Wave propagation Number: $k = 2\pi / \lambda = \omega \sqrt{\mu\varepsilon}$
Previously, we called this β.

NOW let $k = 2\pi / \lambda = \omega \sqrt{\mu\varepsilon}$
And $\beta^2 = k^2 - k_c^2$

Cutoff wavenumber k_c depends on geometry of wave guide.
This will be the wavenumber $k_c = \omega_c \sqrt{\mu\varepsilon}$

ω_c is the cutoff frequency of the waveguide in the mode (TEM/TE/TM) mode of interest. The cutoff frequency is the LOWEST frequency where a given mode will propagate. The mode MAY propagate at frequencies of ω_c or higher.

TEM field solutions
The TEM fields are defined when Ez = Hz = 0.

Substitute into equations 3.5:
BUT we get the result Ex=Ey=Hx=Hy=0 UNLESS $k_c = 0$.
SO, $k_c = 0$ for TEM waves. (This is why we never saw k_c in the transmission line section.)
TEM waves will propagate from DC to high frequencies.

To find TEM field solutions:
Return to Maxwell's Equations and derive Helmholtz wave equation
VIEWGRAPH p.16

Apply to Ex and Ey (and Hx and Hy) components:

Examine TEM modes:

For 2-conductor lines, there can be a TEM mode.
For a closed-conductor line, like a waveguide, there cannot be a TEM mode, since V=0 (metal all around), then Ex=Ey=0, and Ez already =0, so there cannot be TEM modes.

<u>Wave Impedance</u>
$Z_{TEM} = Ex / Hy = \sqrt{\mu / \varepsilon} = \eta = 377.$

This is NOT the Characteristic Impedance of the transmission line (Zo = Vo+ / Io+).

Method of Solution:

1. Solve Laplace's Equation.
   This can be done analytically or numerically. Analytical methods you "guess" the form of the solution, which will have several unknown constants. Example: waveguide will have a sine function, but we don't know the amplitude or number of periods.

2. Use boundary conditions (Φ or $d\Phi/dN$ on surface) to find unknown constants. Now know potential distribution (Φ).

3. Use Φ and

$$V_{12} = \Phi_1 - \Phi_2 = \int_1^2 \overline{E} \bullet \overline{d\ell}$$

   to find **E**

   Find **H** from $\nabla \times \mathbf{E} = -j\omega\mu\, \mathbf{H}$

<u>TE</u> <u>Modes</u> Transverse Electric $E_z = 0$, $H_z \neq 0$

VIEWGRAPH eqns. 3.19

Now, $k_c \neq 0$, so $\beta^2 = k^2 - k_c^2$, function of frequency and guide geometry.

(TE waves cannot propagate at DC)
(TE waves CAN propagate in wave guides, transmission lines, open environment)

<u>Method of solution:</u>

1. Solve Helmholtz equation for either Hz (TE) or Ez (TM). This can be done analytically or numerically. In the analytical case, you guess the form of the solution, which will have several unknown constants (like magnitude, phase, number of cycles)

2. Use 3.19 o 3.23 to find transverse components from Ez or Hz.

3. Solve for the constants from the boundary conditions. In metal boundaries, these are that tangential E and normal H = 0 on the boundary. Now you have Ez or Hz.

4. Use Maxwell's equation to find the other E or H components.

Wave Impedance $Z_{TE} = Ex / Hy = - Ey / Hx = k \ \eta / \beta$

<u>TM waves:</u>

Same as TE, except Transverse Magnetic Hz=0, Ez $\neq$ 0

$Z_{TM} = Ex / Hy = - Ey / Hx = \beta \ \eta / k$

<u>General method of solution:</u>

Given the physical dimensions, materials, etc. Of a guide or line, determine the modes that MAY be excited. Then, design a feed system to get those modes.

1. Find the TEM, TE, and TM modes individually. Each will have a different cut-off frequency.

2. For a given frequency, determine all modes that MAY be present.
3. For a given feed system, find the modes that are compatible, or design a feed system that is compatible with the desired modes.
4. Total modes in a given guide at a given frequency is the sum of modes with cutoff frequencies below the frequency of interest, that also are compatible with the feed system designed.

EXAMPLE : Please read the example of the parallel-plate waveguide.

EXAMPLE: Rectangular wave guide.