Transmission Line Design Handbook
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4.5 COUPLED MICROSTRIP LINES

4.5.1 Edge-Coupled Microstrip Lines

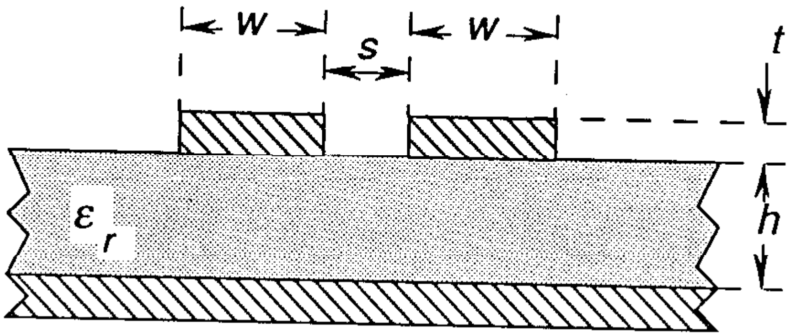

Figure 4.5.1.1: Coupled Microstrip Line

Kirschning and Jansen [8] analyzed this structure with a rigorous spectral-domain hybrid mode calculation. The results of this were then fit numerically. These equations are:

$$Z_{0,e}(0) = \frac{Z_0(0)\sqrt{\dfrac{\varepsilon_{eff}(0)}{\varepsilon_{eff,e}(0)}}}{1.0 - \dfrac{Z_0(0)}{\eta_0}\sqrt{\varepsilon_{eff}(0)}\,Q_4} \tag{4.5.1.1}$$

$$Z_{0,o}(0) = \frac{Z_0(0)\sqrt{\dfrac{\varepsilon_{eff}(0)}{\varepsilon_{eff,o}(0)}}}{1.0 - \dfrac{Z_0(0)}{\eta_0}\sqrt{\varepsilon_{eff}(0)}\,Q_{10}} \tag{4.5.1.2}$$

where $Z_0(0)$ is the zero-frequency impedance of a single, isolated strip (3.5.1.1) and

$$Q_1 = 0.8695\,u^{\,0.194} \tag{4.5.1.3}$$

$$Q_2 = 1.0 + 0.7519\,g + 0.189\,g^{\,2.31} \tag{4.5.1.4}$$

$$Q_3 = 0.1975 + \left[16.6 + (8.4\,/\,g)^{6.0}\right]^{-0.387} + \frac{\ln\left[\dfrac{g^{10}}{1.0 + (g\,/\,3.4)^{10}}\right]}{241} \tag{4.5.1.5}$$

$$Q_4 = \frac{\frac{2\,Q_1}{Q_2}}{e^{-g}\,u^{Q_3} + (2.0 - e^{-g})\,u^{-Q_3}} \tag{4.5.1.6}$$

$$Q_5 = 1.794 + 1.14 \ln\left(1.0 + \frac{0.638}{g + 0.517\,g^{2.43}}\right) \tag{4.5.1.7}$$

$$Q_6 = 0.2305 + \frac{\ln\left[\frac{g^{10}}{1.0 + (g\,/\,5.8)^{10}}\right]}{281.3} + \frac{\ln\left(1.0 + 0.598\,g^{1.154}\right)}{5.1} \tag{4.5.1.8}$$

$$Q_7 = \frac{10.0 + 190.0\,g^2}{1.0 + 82.3\,g^3} \tag{4.5.1.9}$$

$$Q_8 = e^{\left[-6.5 - 0.95\ln(g) - (g\,/\,0.15)^5\right]} \tag{4.5.1.10}$$

$$Q_9 = \ln(Q_7)\,(Q_8 + 1\,/\,16.5) \tag{4.5.1.11}$$

$$Q_{10} = \frac{Q_2 Q_4 - Q_5\,e^{\left[\ln(u)\,Q_6\,u^{-Q_9}\right]}}{Q_2} \tag{4.5.1.12}$$

The frequency-dependent even and odd mode impedances are calculated from the above with:

$$Z_{0,e}(f_n) = \frac{Z_e(0)\left\{0.9408\left[\varepsilon_{eff}(f_n)\right]^{C_e} - 0.9603\right\}^{Q_0}}{\left\{(0.9408 - d_e)\left[\varepsilon_{eff}(0)\right]^{C_e} - 0.9603\right\}^{Q_0}} \tag{4.5.1.13}$$

$$Z_{0,o}(f_n) = Z_0(f_n) + \frac{Z_{0,o}(0)\left[\frac{\varepsilon_{eff,o}(f_n)}{\varepsilon_{eff,o}(0)}\right]^{Q_{22}} - Z_0(f_n)\,Q_{23}}{1.0 + Q_{24} + (0.46\,g)^{2.2}\,Q_{25}} \tag{4.5.1.14}$$

where

$$C_e = 1.0 + 1.275\left[1.0 - e^{-0.004625\,p_e\,\varepsilon_r^{1.674}\left(\frac{f_n}{18.365}\right)^{2.745}}\right] - Q_{12} + Q_{16} - Q_{17} + Q_{18} + Q_{20} \tag{4.5.1.15}$$

$$d_e = 5.086\, q_e \frac{r_e}{0.3838 + 0.386\, q_e} \frac{e^{-22.2\, u^{1.92}}}{1.0 + 1.2992\, r_e} \frac{(\varepsilon_r - 1.0)^6}{1.0 + 10.0\, (\varepsilon_r - 1.0)^6} \tag{4.5.1.16}$$

$$p_e = 4.766\, e^{-3.228\, u^{0.641}} \tag{4.5.1.17}$$

$$q_e = 0.016 + \left(0.0514\, \varepsilon_r\, Q_{21}\right)^{4.524} \tag{4.5.1.18}$$

$$r_e = \left(\frac{f_n}{28.843}\right)^{12} \tag{4.5.1.19}$$

$$Q_{11} = 0.893\left[1.0 - \frac{0.3}{1.0 + 0.7(\varepsilon_r - 1.0)}\right] \tag{4.5.1.20}$$

$$Q_{12} = 2.121\, \frac{(f_n / 20)^{4.91}}{\left[1.0 + Q_{11}\, (f_n / 20)^{4.91}\right]}\, e^{-2.87\, g}\, g^{0.902} \tag{4.5.1.21}$$

$$Q_{13} = 1.0 + 0.038\, (\varepsilon_r / 8)^{5.1} \tag{4.5.1.22}$$

$$Q_{14} = 1.0 + \frac{1.203\, (\varepsilon_r / 15)^4}{1.0 + (\varepsilon_r / 15)^4} \tag{4.5.1.23}$$

$$Q_{15} = \frac{1.887\, e^{-1.5\, g^{0.84}}\, g^{Q_{14}}}{1.0 + \dfrac{0.41\, (f_n / 15)^3\, u^{(2 / Q_{13})}}{0.125 + u^{(1.626 / Q_{13})}}} \tag{4.5.1.24}$$

$$Q_{16} = Q_{15}\left[1.0 + \frac{9.0}{1.0 + 0.403\, (\varepsilon_r - 1.0)^2}\right] \tag{4.5.1.25}$$

$$Q_{17} = 0.394\left[1.0 - e^{-1.47\, (u / 7)^{0.672}}\right]\left[1.0 - e^{-4.25\, (f_n / 20)^{1.87}}\right] \tag{4.5.1.26}$$

$$Q_{18} = 0.61\, \frac{1.0 - e^{-2.13\, (u / 8)^{1.593}}}{1.0 + 6.544\, g^{4.17}} \tag{4.5.1.27}$$

$$Q_{19} = \frac{0.21\, g^4}{\left(1.0 + 0.18\, g^{4.9}\right)\left(1.0 + 0.1\, u^2\right)\left[1.0 + (f_n / 24.0)^3\right]} \tag{4.5.1.28}$$

$$Q_{20} = Q_{19}\left[0.09 + \frac{1.0}{1.0 + 0.1\,(\varepsilon_r - 1.0)^{2.7}}\right] \tag{4.5.1.29}$$

$$Q_{21} = \left|1.0 - \frac{42.54\, g^{0.133}\, e^{-0.812\, g}\, u^{2.5}}{1.0 + 0.033\, u^{2.5}}\right| \tag{4.5.1.30}$$

$$Q_{22} = \frac{0.925\,(f_n / Q_{26})^{1.536}}{1.0 + 0.3\,(f_n / 30)^{1.536}} \tag{4.5.1.31}$$

$$Q_{23} = 1.0 + \frac{0.005\, f_n\, Q_{27}}{\left[1.0 + 0.812\,(f_n / 15.0)^{1.9}\right]\left(1.0 + 0.025\, u^2\right)} \tag{4.5.1.32}$$

$$Q_{24} = 2.506\, Q_{28}\, u^{0.894}\, \frac{\left[\frac{(1.0 + 1.3\, u)\, f_n}{99.25}\right]^{4.29}}{3.575 + u^{0.894}} \tag{4.5.1.33}$$

$$Q_{25} = \left(\frac{0.3\, f_n^2}{10.0 + f_n^2}\right)\left[1.0 + \frac{2.333\,(\varepsilon_r - 1.0)^2}{5.0 + (\varepsilon_r - 1.0)^2}\right] \tag{4.5.1.34}$$

$$Q_{26} = 30.0 - \frac{22.2\left[\frac{(\varepsilon_r - 1.0)}{13}\right]^{12}}{1.0 + 3.0\left[\frac{(\varepsilon_r - 1.0)}{13}\right]^{12}} - Q_{29} \tag{4.5.1.35}$$

$$Q_{27} = 0.4\, g^{0.84}\left[1.0 + \frac{2.5\,(\varepsilon_r - 1.0)^{1.5}}{5.0 + (\varepsilon_r - 1.0)^{1.5}}\right] \tag{4.5.1.36}$$

$$Q_{28} = \frac{0.149\,(\varepsilon_r - 1.0)^3}{94.5 + 0.038\,(\varepsilon_r - 1.0)^3} \tag{4.5.1.37}$$

$$Q_{29} = \frac{15.16}{1.0 + 0.196\,(\varepsilon_r - 1.0)^2} \tag{4.5.1.38}$$

$$u = w / h \tag{4.5.1.39}$$

$$g = s / h \tag{4.5.1.40}$$

$$f_n = f\,h \tag{4.5.1.41}$$

where f is the frequency in GHz and h is the substrate thickness in mm. The effective dielectric constants for the even and odd modes are calculated by beginning with the dc ε_{eff} for a single strip derived in [5]:

$$\varepsilon_{eff}(0) = 0.5\,(\varepsilon_r + 1.0) + 0.5\,(\varepsilon_r - 1.0)\left(1.0 + \frac{10.0}{v}\right)^{-a} e^{v}\, b_e(\varepsilon_r) \tag{4.5.1.42}$$

where

$$v = \frac{u\,(20.0 + g^2)}{10.0 + g^2} + g\,e^{-g} \tag{4.5.1.43}$$

$$a_e(v) = 1.0 + \frac{\ln\left[\dfrac{v^4 + (v\,/\,52.0)^2}{v^4 + 0.432}\right]}{49.0} + \frac{\ln\left[1.0 + (v\,/\,18.1)^3\right]}{18.7} \tag{4.5.1.44}$$

$$b_e(\varepsilon_r) = 0.564\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3.0}\right)^{0.053} \tag{4.5.1.45}$$

The even and odd mode effective relative dielectric constants at dc are then calculated:

$$\varepsilon_{eff,o}(0) = \left[0.5\,(\varepsilon_r + 1.0) + a_0(u,\,\varepsilon_r) - \varepsilon_{eff}(0)\right] e^{-c_0\, g^{d_0}} + \varepsilon_{eff}(0) \tag{4.5.1.46}$$

$$\varepsilon_{eff,e}(0) = 0.5\,(\varepsilon_r + 1.0) + 0.5\,(\varepsilon_r - 1.0)\left[1.0 + \left(\frac{10.0}{v}\right)^{-a_e(v)\, b_e(\varepsilon_r)}\right] \tag{4.5.1.47}$$

where

$$a_0(u,\,\varepsilon_r) = 0.7287\left[\varepsilon_{eff}(0) - 0.5\,(\varepsilon_r + 1.0)\right](1.0 - e^{-0.179\,u}) \tag{4.5.1.48}$$

$$b_0(\varepsilon_r) = \frac{0.747\,\varepsilon_r}{0.15 + \varepsilon_r} \tag{4.5.1.49}$$

$$c_0 = b_0(\varepsilon_r) - \left[b_0(\varepsilon_r) - 0.207\right] e^{-0.414\,u} \tag{4.5.1.50}$$

$$d_0 = 0.593 + 0.694\,e^{-0.562\,u} \tag{4.5.1.51}$$

These can be corrected for frequency-dependent dispersion with:

$$\varepsilon_{eff,o}(f_n) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff,o}(0)}{1.0 + F_o(f_n)} \tag{4.5.1.52}$$

$$\varepsilon_{eff,e}(f_n) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff,e}(0)}{1.0 + F_e(f_n)} \tag{4.5.1.53}$$

where

$$F_o(f_n) = P_1 P_2 [(P_3 P_4 + 0.1844) f_n P_{15}]^{1.5763} \tag{4.5.1.54}$$

$$F_e(f_n) = P_1 P_2 [(P_3 P_4 + 0.1844 P_7) f_n]^{1.5763} \tag{4.5.1.55}$$

$$P_1 = 0.27488 + \left[0.6315 + \frac{0.525}{(1.0 + 0.0157 f_n)^{20}}\right] u - 0.065683 \, e^{-8.7513 u} \tag{4.5.1.56}$$

$$P_2 = 0.33622 \left(1.0 - e^{-0.03442 \varepsilon_r}\right) \tag{4.5.1.57}$$

$$P_3 = 0.0363 \, e^{-4.6 u} \left[1 - e^{-(f_n / 38.7)^{4.97}}\right] \tag{4.5.1.58}$$

$$P_4 = 1.0 + 2.751 \left[1.0 - e^{(-\varepsilon_r / 15.916)^8}\right] \tag{4.5.1.59}$$

$$P_5 = 0.334 \, e^{-3.3(\varepsilon_r / 15)^3} + 0.746 \tag{4.5.1.60}$$

$$P_6 = P_5 \, e^{-(f_n / 18)^{0.368}} \tag{4.5.1.61}$$

$$P_7 = 1.0 + 4.069 P_6 \, g^{0.479} \, e^{(-1.347 g^{0.595} - 0.17 g^{2.5})} \tag{4.5.1.62}$$

$$P_8 = 0.7168 \left[1.0 + \frac{1.076}{1.0 + .0576 (\varepsilon_r - 1.0)}\right] \tag{4.5.1.63}$$

$$P_9 = P_8 - 0.7913 \left[1.0 - e^{-(f_n / 20)^{1.424}}\right] \tan^{-1}\left[2.481 (\varepsilon_r / 8)^{0.946}\right] \tag{4.5.1.64}$$

$$P_{10} = 0.242 (\varepsilon_r - 1.0)^{0.55} \tag{4.5.1.65}$$

$$P_{11} = 0.6366 \left(e^{-3.401 f_n} - 1.0\right) \tan^{-1}\left[1.263 (u / 3)^{1.629}\right] \tag{4.5.1.66}$$

$$P_{12} = P_9 + \frac{1.0 - P_9}{1.0 + 1.183\ u^{1.376}} \tag{4.5.1.67}$$

$$P_{13} = \frac{1.695\ P_{10}}{0.414 + 1.605\ P_{10}} \tag{4.5.1.68}$$

$$P_{14} = 0.8928 + 0.10722 \left[1.0 - e^{\ 0.42(f_n / 20.0)^{3.215}}\right] \tag{4.5.1.69}$$

$$P_{15} = \left| 1.0 - \frac{0.8928\ (1.0 + P_{11})\ P_{12}\ e^{-P_{13}\ g^{1.092}}}{P_{14}} \right| \tag{4.5.1.70}$$

Equations are valid for:

$0.1 \leq w / h \leq 10.0$

$0.1 \leq s / h \leq 10.0$

$1.0 \leq \varepsilon_r \leq 18$

and the stated accuracy for the range $\varepsilon_r \leq 12.9$ and $f_n \leq 15$ is better than 1.5%.

The above equations are lengthy; they can be simplified to their dispersionless form in many applications. PCB dimensions are not usually practical for less than about 6 dB of coupling (see the section on Lange couplers for tighter couplings). Other coupling structures (*i.e.*, branch-line hybrids, Lange couplers, Wilkinson dividers, and directional couplers) can be used when tighter or wideband coupling is required.

REFERENCES

[1] Bochtler, U. and F. Endress, "CAD Program Designs Stripline Couplers," *Microwaves & RF*, December 1986, pp. 91–95.

[2] Bryant, T.G., and J.A. Weiss, "Parameters of Microstrip Transmission Lines and of Coupled Pairs of Microstrip Lines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-16, December 1968, pp. 1021–1027.

[3] Garg, Ramesh, and I.J. Bahl, "Characteristics of Coupled Microstriplines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-27, No. 7, July 1979, pp. 700–705 and "Correction to 'Characteristics of Coupled Microstriplines,'" Vol. MTT-28, No. 3, March 1980, p. 272.

Getsinger, W.J., "Dispersion of Parallel-Coupled Microstrip," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-23, No. 3, March 1973, pp. 144–145.

[5] Hammerstad, E., and Ø. Jensen, "Accurate Models for Microstrip Computer-Aided Design," *1980 IEEE MTT-S International Microwave Symposium Digest*, pp. 407–409.

[6] Hinton, J.H., "On Design of Coupled Microstrip Lines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-28, No. 3, March 1980, pp. 272.

[7] Horno, Manuel, and Ricardo Marqués, "Coupled Microstrips on Double Anisotropic Layers," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 32, No. 4, April 1984, pp. 467–470.

[8] Kirschning, Manford, and Rolf H. Jansen, "Accurate Wide-Range Design Equations for the Frequency-Dependent Characteristic of Parallel Coupled Microstrip Lines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-32, No. 1, January 1984, pp. 83–90.

[9] Krage, Mark K., and George I. Haddad, "Characteristics of Coupled Microstrip Transmission Lines-I: Coupled Mode Formulation of Inhomogeneous Line," *IEEE Transactions on Microwave Theory and Techniques*, MTT-18, No. 4, April 1970, pp. 217-222.

[10] Krage, Mark K., and George I. Haddad, "Characteristics of Coupled Microstrip Transmission Lines-II: Evaluation of Coupled-Line Parameters," *IEEE Transactions on Microwave Theory and Techniques*, MTT-18, No. 4, April 1970, pp. 222-228.

[11] Lau, Wai Yuen, "Network Analysis Verifies Models in CAD Packages," *Microwaves & RF*, November 1989, pp. 99–110.

[12] Lev, James J., "Calculator Program Synthesizes Microstrip Coupled Lines," *Microwaves & RF*, May 1984, pp. 215–220.

[13] March, Steven, "Microstrip Packaging: Watch the Last Step," *Microwaves*, December 1981, pp. 83–94.

[14] Mirshekar-Syahkal, D., "An Accurate Determination of Dielectric Loss Effect in Monolithic Microwave Integrated Circuits Including Microstrip and Coupled Microstrip Lines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-31, No. 11, November 1983, pp. 950–954.

[15] Quaglia, Antonio, "Use Coupled Lines to Achieve High Impedances in Microstrip," *Microwaves & RF*, October 1983, pp. 120–127.

[16] Rao, B. Rama, "Effect of Loss and Frequency Dispersion on the Performance of Microstrip Directional Couplers and Coupled Line Filters," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 22, No. 8, July 1974, p. 747–750.

[17] Rosloniec, Stanislaw, "Design of Coupled Microstrip Lines by Optimization Methods," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-35, No. 11, November 1987, pp. 1072–1074.

[18] Tripathi, V.K., "Loss Calculations for Coupled Transmission-Line Structures," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-20, No. 2, February 1972, pp. 178–180.

[19] Zehentner, Jan, "Analysis and Synthesis of Coupled Microstrip Lines by Polynomials," *Microwave Journal*, May 1980, pp. 95–110.

4.5.2 Asymmetric Coupled Microstrip Line

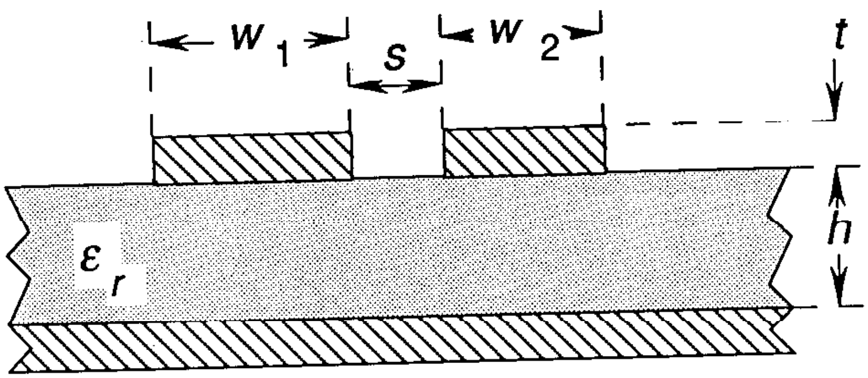

Figure 4.5.2.1: Asymmetric Coupled Microstrip Lines

This configuration has received a great deal of attenuation for its potential applications in filters, etc. Current designs are based on symmetric couplers and the asymmetry would give us an additional degree of freedom in design. Asymmetric couplers allow the coupling of power at the same time as an impedance transformation takes place.

Unfortunately, closed-form equations for microstrip line do not exist. The reader with sufficient skill may be able to adapt the curves or analyses presented in the references. EM programs may also be used.

The general relationships for coupling and impedances in asymmetric couplers can be defined [1]

$$Z_{0,e}{}^{a} = \frac{\sqrt{1.0 - k^2}}{G_a - k\sqrt{G_a G_b}} \tag{4.5.2.1}$$

$$Z_{0,o}{}^{a} = \frac{\sqrt{1.0 - k^2}}{G_a - k\sqrt{G_a G_b}} \tag{4.5.2.2}$$

$$Z_{0,e}{}^{b} = \frac{\sqrt{1.0 - k^2}}{G_a - k\sqrt{G_a G_b}} \tag{4.5.2.3}$$