ECE 6130 Quadrature Coupler

Text Section 7.5

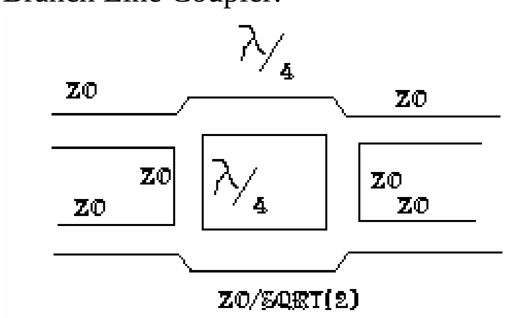
What is a quadrature coupler and how does it work?

Quadrature Coupler

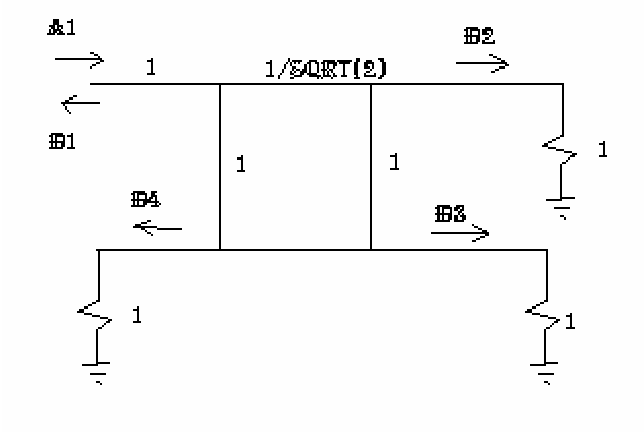
90 degree phase shift between output ports.
Power Evenly divided (-3dB) between output ports.
All ports matched.
No coupling to port 4 (complete isolation).

$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

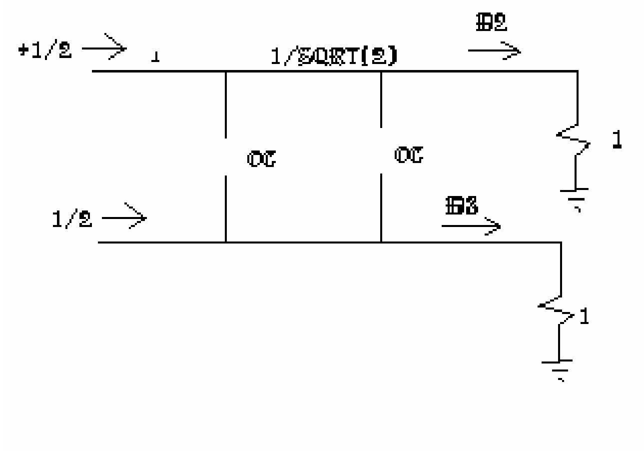
Branch Line Coupler:



Even-Odd Mode Analysis:
Normalized Impedance Circuit Model:



EVEN MODE:



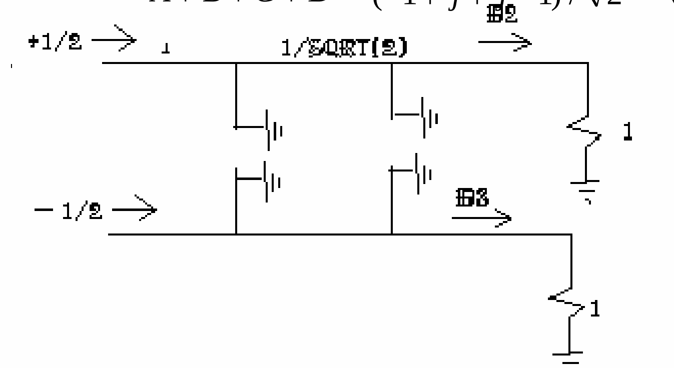
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = [\lambda/8\ Stub][\lambda/4\ TL][\lambda/8\ Stub]$$

$$= \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} \cos\beta\ell & jZo\sin\beta\ell \\ jZo\sin\beta\ell & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & c0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

$$S_{11} = \Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{(-1+j-j+1)/\sqrt{2}}{(-1+j+j-1)/\sqrt{2}} = 0$$

$$S_{21} = T_e = \frac{2}{A+B+C+D} = \frac{2}{(-1+j+j-1)/\sqrt{2}} = \frac{-1}{\sqrt{2}}(1+j)$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = [\lambda/8\ Stub][\lambda/4\ TL][\lambda/8\ Stub]$$

$$= \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} \cos\beta\ell & jZo\sin\beta\ell \\ jZo\sin\beta\ell & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & c0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$S_{11} = \Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{(1+j-j-1)/\sqrt{2}}{(1+j+j+1)/\sqrt{2}} = 0$$

$$S_{21} = T_e = \frac{2}{A+B+C+D} = \frac{2}{(1+j+j+1)/\sqrt{2}} = \frac{1}{\sqrt{2}}(1-j)$$

COMBINE EVEN and ODD MODES:

Incident voltage at node 1: $V_{1e} = V_{1o} = 1/2$
Incident voltage at node 4: $V_{4e} = - V_{4o} = -1/2$

Amplitudes of waves exiting each node:

$B_1 = V_{1e}\,\Gamma_e + V_{1o}\,\Gamma_o = \Gamma_e/2 + \Gamma_o/2 = 0$ (Port 1 is matched)
$B_2 = V_{1e}\,T_e + V_{1o}\,T_o = T_e/2 + T_o/2 = - j / \text{sqrt}(2)$ (Half power, -90 degree phase shift from port 1 to 2)
$B_3 = V_{4e}\,T_e + V_{4o}\,T_o = T_e/2 - T_o/2 = -1/\text{sqrt}(2)$ (Half power, 180 degree phase shift from port 1 to 3)
$B_4 = V_{4e}\,\Gamma_e + V_{4o}\,\Gamma_o = \Gamma_e/2 - \Gamma_o/2 = 0$ (Port 4 is isolated)

This is the first row of the S matrix:

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Other rows are found by transposition:

Row 2 for example:

1 → 2 =0
2 → 1 =j
3 → 4 =1
4→ 3 =0