Advanced Forward Methods for Complex Wire Fault Modeling

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Abstract—This paper presents novel implementation of forward modeling methods for simulating reflectometry responses of faults in the shields of coaxial cable or other shielded lines. First, cross-sectional modeling was used to determine the characteristic impedance of various wire sections. These values were then incorporated into longitudinal models to simulate the overall reflectometry response. The finite-difference method (FDM) is a cross-sectional modeling method that was used to simulate cross-sectional characteristic impedance. Results using this method are accurate within 1% of analytical solutions, and can be simulated very quickly using in-house codes. The finite-integral technique (FIT) was also used to model chafes on wires with TEM and higher order modes. Because FIT is computationally slow, a curve fitting technique was used to predict the chafe profile within 0.01% of the simulated values. Modified transmission matrix (MTM) and S-parameter methods were used to provide responses with accuracies within 0.3% of the measured profiles.

I. INTRODUCTION AND BACKGROUND

Location and diagnosis of faults in aging electrical wiring can enable their timely repair, thus preventing costly and potentially hazardous failures. This research focuses on one of the most challenging problems in electrical fault location—finding small chafes in the shields of shielded wires (coax, twisted shielded pair, etc.). These small faults produce such small reflection signatures that in many cases they are undetectable against the background noise in aircraft and spacecraft [1]. The goal of this work is to use the models developed here to design new detection schemes and predict when small fault detection may be possible.

Hard faults (opens/shorts) have been well studied [2]. These faults are easier to find, and traditional reflectometry measurements are effective for locating them. However, partial faults, or chafes, are more difficult to identify and model because the fault signatures are small and the electrical system usually does not show any noticeable symptom until the fault is severe. Chafes are typically the result of abrasion or vibration against other wire or structural members, and they often worsen over time. Like human health, early detection is key to curing the problem and preventing catastrophic disasters.

The objective of this wire modeling is to simulate the reflectometry response of the wiring system. Reflectometry is a method of determining wire characteristics from reflections of high frequency electrical signals resulting from impedance discontinuities. Finding the small anomalies of frayed wire before they become hard faults is of significant interest, yet a challenging problem. These types of faults have been far less studied than hard faults, and the forward modeling methods described in this paper present novel implementation of these methods specialized to the chafe problem. Chafing insulation results in a very small change in the wire impedance, and because the reflection depends on the magnitude of the impedance discontinuity, this results in a very small reflection that may be lost in the noise of the environment and measurements. Therefore, the problem has previously been considered more difficult to solve relative to the impact of hard faults. Fortunately, chafes are much more detectable in shielded wiring, where noise levels are significantly lower and impedance levels remain more consistent along the wire length, and where controlled impedances are less affected by surrounding structures, environment, and vibration [3]. Models and analysis of shielded cables, where the external environment has little or no impact on the cable, enable location of much smaller faults than previously detectable.

In modeling these wiring systems, two types of forward models are used together: cross-sectional and longitudinal. Cross-sectional models are used to determine characteristic impedance of wire sections, and these impedances are then implemented longitudinally where an overall system response can be obtained using reflectometry. These two modeling processes are illustrated in Figure 1.

This paper provides detailed models of shielded faults and a method to integrate fault models (including measured data) in a unified forward model that describes the effects of the fault in a realistic electrical system. Unique aspects of this
model include its modularity (ability to efficiently integrate data from multiple simulations and measurements), detailed fault models (including frequency dependence of the faults), and the ability to model small faults with great precision while still incorporating them into a full system model (which normally has lower precision for more efficient computation).

II. CROSS-SECTIONAL MODELING METHODS

A. Finite-Difference Method (FDM)

The finite-difference method [4] is a numerical tool for solving the generalized Poisson equation,

$$\nabla \cdot (\epsilon_r \nabla V) = -\frac{\rho}{\epsilon_0},$$

(1)

where $V$ is the unknown voltage potential function, $\rho$ is the charge density function, $\epsilon_r$ is the relative permittivity function, and $\epsilon_0$ is the permittivity of free-space. The basic principle of FDM is to approximate the Poisson equation by replacing derivatives by finite-differences and to sample the continuous functions along a discrete, finite grid. The net result is a linear system of equations that may be solved to find the potential fields and characteristic impedance of the wire.

The fault impedance $Z_F$ is found by using techniques outlined in [5], [6] and [7]. First, a 2D cross-section of damaged cable is modeled by defining the appropriate dielectric function $\epsilon_r$ and boundary conditions for $V$. An example model is shown in Figure 2, which depicts a pie-cake cutaway of $\phi = 36^\circ$ on a slice of RG-58 C/U coaxial cable. Such a model is then solved via FDM to find potentials $V$, after which a set of electric field samples may be found by applying $E = -\nabla V$. A simple method for this procedure is also outlined in [4], which again uses finite-differences as an approximation to the gradient operation. Finally, the total charge $q$ per unit length along the inner conductor is found by evaluating Gauss’s law. In integral form, this is written as

$$\epsilon_0 \int_S \epsilon_r(x,y) E(x,y) \cdot dn = q,$$

(2)

where $dn$ is the differential normal vector that points outward along the closed surface $S$, which in 2D is simply a closed contour. Because $E$ and $\epsilon_r$ are both discretely sampled along a rectangular grid, evaluation of $q$ may be readily accomplished through the use of a finite summation along the appropriate contour. Any choice for $S$ is acceptable, provided that it completely encloses the inner conductor of the model and fits within the shield. It is therefore common to simply define $S$ as a rectangle, as shown by the box surrounding the center conductor in Figure 2.

Once the Gaussian contour has been integrated to find $q$, the next step is to calculate the capacitance $C$ per unit length of the FDM model. We therefore begin by noting that

$$C = \frac{q}{V_0},$$

(3)

where $V_0$ is the excitation voltage of the inner conductor. This value is arbitrarily defined as a boundary condition within the FDM model, and so may be simply fixed at a normalized value of $V_0 = 1.0 \text{ V}$.

With $C$ now a known value, we are ready to express the characteristic impedance $Z_F$ using

$$Z_F = \sqrt{\frac{L}{C}},$$

(4)

where $L$ is the inductance per unit length. Although $L$ is an unknown value, it is possible to sidestep this parameter by noting that the velocity of propagation $u_p$ for a TEM transmission line satisfies

$$u_p = \frac{1}{\sqrt{LC}}.$$  

(5)

In the absence of any dielectric insulation between the inner and outer conductors, it is a known fact that $u_p = c_0 = 2.996 \times 10^8 \text{ m/s}$, which is the speed of light in a vacuum [8]. Furthermore, because $L$ is independent of any non-magnetic materials within the system, it is possible to write

$$c_0 = \frac{1}{\sqrt{LC_0}},$$

(6)

where $C_0$ is the capacitance per unit length in the absence of any insulation. With this information in mind, we may now express $Z_F$ in a modified form as

$$Z_F = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC_0}{CC_0}} = \frac{1}{c_0\sqrt{CC_0}}.$$  

(7)

As we can see, $Z_F$ is now independent of any inductance term $L$ and the only new information we need is $C_0$. Fortunately, this value may be readily computed by simulating an identical system as before, but without any embedded insulation. For comparison, the characteristic impedance of an ideal coaxial cable can then be calculated analytically [8], and demonstrates less than 1% error against the FDM simulations. We therefore have a simple, accurate method for computing $Z_F$ through the use of FDM.

The cable can be modeled using this method by programming the cable dimensions, along with the fault size, in a 2D grid as shown in Figure 2. The box surrounding the
center conductor will then be used to calculate the contour integral from the voltage potential matrix, eventually yielding the characteristic impedance of the chafe through the process described in this section. Results for this method compared to other methods are found in Table I, where RG-58 was used as the wire model. Measurement of undamaged cable is possible simply by matching the cable end while connected to a reflectometry scope. The load on the wire end can be adjusted until the reflections become zero, thus matching the cable and determining its characteristic impedance. Measurement of the characteristic impedance of a single small chafe is much more difficult, and thus methods of validated computational forward modeling become rather useful. The analytical solutions for determining characteristic impedance of coaxial cable have been well studied and can be found in [8].

### TABLE I

<table>
<thead>
<tr>
<th>Method</th>
<th>$Z_0$</th>
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</thead>
<tbody>
<tr>
<td>FDM</td>
<td>51.4Ω</td>
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<tr>
<td>FIT</td>
<td>50.0Ω</td>
</tr>
<tr>
<td>Analytical</td>
<td>50.9Ω</td>
</tr>
<tr>
<td>Measured</td>
<td>50.0Ω</td>
</tr>
</tbody>
</table>

B. Finite-Integral Technique (FIT)

The finite-integral technique (FIT) is a method that numerically solves electromagnetic field problems in the spatial and frequency domains [9]. FDM provides the impedance of the chafe on a 2D cross-section for a TEM wire. However, when the shield is damaged as shown in Figure 4, the field lines bend and the models are no longer strictly TEM. Thus, FIT can be used to find the cross-sectional 2D characteristic impedance of a 3D chafe, including the higher order modes developed when the fields exit the chafe. Although this makes FIT more computationally expensive, it also allows one to model the effects of more complex faults at specific, higher frequencies. FIT is therefore a popular method used in commercial software packages such as Computer Simulation Technology (CST) [10].

Figure 4 shows the output of an FIT simulation for a 60° cutaway in RG-58 C/U coaxial cable at 5 GHz. Because FIT can be computationally expensive at high resolution, we can utilize a polynomial curve fitting algorithm to minimize the points required. This is demonstrated by the impedance profile shown in Figure 5.

Like most iteration-based algorithms, FIT can be computationally expensive at high resolution or where the point of interest is small and the wire is long. While FDM can be run in a matter of seconds, FIT simulations can take several hours. Efficiency and precision can be critical in modeling, yet they are often mutually exclusive. Instead of running the numerical modeling for every single chafe possibility, we can utilize a polynomial curve fitting algorithm to minimize the detailed simulations required.

The quasi-TEM mode simulation using FIT combines the effect of both TEM and higher order modes. This method was used to calculate the characteristic impedance of chafed cable. Polynomial curve fitting was used to represent the impedance profile of the faulty coax, as in Figure 5.

This method yielded the characteristic impedance of chafe by programming the dimensions of the cable and the chafe in CST software, which then computes the cross-sectional impedance. Results for this method are found in Figure 5, and a comparison this method is found in Table I.
III. LONGITUDINAL MODELING METHODS

Once the characteristic impedance of the chafe is obtained using FDM, FIT, or perhaps measurements, a longitudinal modeling method can be used to simulate the overall forward response. These methods simulate the time-domain reflectometry (TDR) response. Although the step-function TDR is presented, these methods can also be applied to pulse-shaped TDR, spectral time-domain reflectometry (STDR), or spread-spectrum time-domain reflectometry (SSTDR) [11]. The only difference in the simulation process is the multiplication of the different source signal in the frequency domain before performing the inverse Fourier transform.

A. Finite-Difference Time-Domain (FDTD) Method

The finite-difference time-domain (FDTD) method is a computational electrodynamics modeling technique that solves the differential form of the telegrapher’s equations in the time domain. These equations are defined as

\[
\begin{align*}
\frac{\partial V(z,t)}{\partial z} &= R(z)I(z,t) + L(z)\frac{\partial I(z,t)}{\partial t}, \quad (8) \\
\frac{\partial I(z,t)}{\partial z} &= G(z)V(z,t) + C(z)\frac{\partial V(z,t)}{\partial t}, \quad (9)
\end{align*}
\]

where \( R \) (resistance), \( L \) (inductance), \( G \) (shunt conductance), and \( C \) (capacitance) are the wire or fault parameters, which can be defined either for the wire as a whole or changed per cell in the simulation. The voltages and currents at any point along the line are simulated, including reflected voltages from impedance discontinuities caused by wire chafing. These reflections, along with the input signal, can then be used to calculate the reflectometry response [12].

One advantage of the FDTD method is that it can be used for simulation of faults containing graded (gradual) impedance changes along the line. Many faults contain graded changes, so these simulations provide a more realistic method of determining the types of reflections and signal changes that can be expected from such faults. A discretized approach to adjusting the RLGC parameters can be implemented in a cell-by-cell manner, where the resulting characteristic impedance gradually changes across the length of the fault. FDTD can then be used to obtain reflectometry responses for chafes and other impedance discontinuities by simulating the reflected voltages and taking the corresponding impulse signal into account. These voltages can be obtained at any point along the line and source signals can be modified within the software. An example of the transmitted and reflected signals is shown in Figure 6. Calculations can be done in either the time or frequency domain with any type of input signal.

B. Modified Transmission Matrix (MTM) Method

The transmission matrix method, which evaluates ABCD matrices [13], is used to evaluate linear networks and is commonly used in microwave engineering [14]. A modified version of this method was used to simulate the reflectometry response of cascaded transmission lines, as shown in the simple TDR setup displayed at the top of Figure 7. This is called the modified transmission matrix (MTM) method. A TDR tester, with characteristic impedance \( Z_S \), is connected to the transmission line as a signal source. A load, with characteristic impedance \( Z_L \), is at the end of wire. The transmission line has a characteristic impedance \( Z_T \), phase constant \( \beta \), and length \( l \). The equivalent circuit is depicted at the bottom of Figure 7.

The source \( (M_1) \), lossless transmission line \( (M_2) \) and the load \( (M_3) \) can be represented in ABCD matrices as:

\[
M_1 = \begin{bmatrix} 1 & Z_S \\ 0 & 1 \end{bmatrix}
\]

\[
M_2 = \begin{bmatrix} \cos \beta l & jZ_T \sin \beta l \\ jY_L \sin \beta l & \cos \beta l \end{bmatrix}
\]

\[
M_3 = \begin{bmatrix} 1 & 0 \\ 1/Z_L & 1 \end{bmatrix}
\]

The consolidated matrix becomes:
\[ M = M_1 M_2 M_3 \]  \hspace{1cm} (13)

\[ M_2 = \begin{bmatrix} \cosh \gamma l & Z_T \sinh \gamma l \\ Y_T \sinh \gamma l & \cosh \gamma l \end{bmatrix} \]  \hspace{1cm} (15)

where \( \xi \) represents the unnecessary parameters that can be discarded in order to conserve computational expense. For a lossy transmission line, \( M_2 \) can be written as

\[ M_2 = \begin{bmatrix} \cosh \gamma l & Z_T \sinh \gamma l \\ Y_T \sinh \gamma l & \cosh \gamma l \end{bmatrix} \]  \hspace{1cm} (15)

where the complex propagation constant \( \gamma = \alpha + j \beta \) and the attenuation constant \( \alpha \) is nonzero.

To use the MTM method for TDR, we need to consider the wave propagation of the forward and reflective paths. Figure 8 shows an \( n \)-section configuration. TDR data is typically acquired between source (\( M_1 \)) and the first section of the wire (\( M_2 \)). The TDR transfer function is essentially the relationship or ratio of voltages \( V_1 \) (incident) and \( V_2 \) (reflected).

The forward path \( V_1 \) and reverse path \( V_2 \) are represented by:

\[ V_1 = \prod_{x=1}^{n} M_x \cdot V_n \]  \hspace{1cm} (16)

\[ V_2 = \prod_{x=2}^{n} M_x \cdot V_n \]  \hspace{1cm} (17)

where \( M_A \) denotes the element \( A \) of the ABCD matrix \( M \).

The transfer function is calculated as,

\[ H(\omega) = \frac{V_2}{V_1} = \prod_{x=2}^{n} M_x / \prod_{x=1}^{n} M_x \]  \hspace{1cm} (18)

and the time domain response is then simply the inverse Fourier transform,

\[ \Gamma(t) = \mathcal{F}^{-1} \{ S(\omega)H(\omega) \} \]  \hspace{1cm} (19)

where \( S(\omega) \) is the source signal in frequency domain and \( H(\omega) \) is the transfer function of the TDR. Figure 9 shows a multi-section setup with a reactive load. The result is shown in Figure 10, showing good agreement between simulated and measured values. Although the step-function TDR is presented, this method can also be applied to pulse-shaped TDR, STDR or SSTDR. The only difference in the simulation process is the multiplication of the different source signal in the frequency domain before performing the inverse Fourier transform.

**Integration of FIT and MTM:** The MTM and FIT methods were combined to predict the TDR signature for a chafed RG58 coax, as shown in Figure 11. A Campbell Scientific TDR100 is used as the test source. A shield cutaway of 120°, 5 cm in length, located at 6.5 ft on a 12 ft RG-58 coaxial cable was synthesized. This stepped voltage TDR produced a small reflected pulse shown in Figure 12. This is because the chafe creates two overlapping reflections—one at the start of the chafe and another, nearly equal and opposite, at the end of the chafe.

Instead of discretizing the wire into numerous FDTD grids, the MTM method simply represents the entire structure with three sets of ABCD matrices, which represent the section before the chafe, the chafe itself and the section after. In order to have a functional and realistic forward model, the frequency-dependent characteristic impedance of the chafed wire section is obtained using this proposed method.

The TDR result of the chafed scenario described previously is presented in Figure 12. The measured and simulated results agreed excellently at the location of interest. If this simulation
was done entirely using 3D FIT, it could take several hours to complete depending on the resolution setup. With the defined fault profile and the assistance of the frequency-domain MTM method, the proposed method took less than a second to perform the same task. Additionally, with the defined fault profile, one can easily plot the prediction of 5 cm chafes of various angle cutaways on an RG-58 at 6.5 ft. This is shown in Figure 13.

With the integration of cross-sectional and longitudinal models, the modeling of chafed wires can be made more efficiently.

C. S-Parameter Method

Another longitudinal method is the S-parameter approach [15]. This method differs from the MTM method in that matched load conditions are used to determine the matrix parameters, rather than open/short conditions. The S-parameters are then defined by transmitted/reflected waves and reflection coefficients.

In order to simulate the response of the wire system (the forward model), a system of S-parameter equations was derived for the damaged wire case. This case included one chafed section of length $z_2$, located at a distance $z_1$ along a wire of total length $z_T$. This is shown in Figure 14.

The transfer function $H(\omega)$ was derived from S-parameter theory using a highly detailed model [15]. The forward voltage $V_M(\omega)$ can then be obtained by multiplying the frequency response $V_S(\omega)$ of the input (source) signal with the transfer function $H(\omega)$ of the system. Time-dependent voltage $v_M(t)$ is obtained by using the inverse Fourier transform. The following equations outline the steps taken to obtain $v_M(t)$, the simulated time-domain reflectometry (TDR) response:

$$V_S(\omega) = \mathcal{F}\{v_S(t)\}$$  \hspace{1cm} (20)

$$V_M(\omega) = H(\omega) \cdot V_S(\omega)$$ \hspace{1cm} (21)

$$v_M(t) = \mathcal{F}^{-1}\{V_M(\omega)\}$$ \hspace{1cm} (22)

The resulting system response from (22) can then be plotted as shown in Figure 15. This method was validated to be accurate within 0.3% of measured impedance profiles.

| Method       | $|\Gamma|$ |
|--------------|-----------|
| FDTD         | 0.01      |
| MTM          | 0.01      |
| S-parameters | 0.003     |

### IV. Conclusion

This paper presents novel implementation of forward methods used for simulating faults in the shields of coaxial cable and other shielded lines. First, cross-sectional modeling was
used to determine the characteristic impedance of various wire sections, and then longitudinal methods were used to simulate the reflectometry response using these characteristic impedance values.

The finite-difference method (FDM) is a cross-sectional modeling method that was used to simulate cross-sectional characteristic impedance. Results using this method are accurate within 1% of the analytical solution for characteristic impedance of a coaxial cable and can be simulated rather quickly using an efficient algorithm which was developed.

Because chafed wire carries a mix of TEM and higher order modes, the finite-integral technique (FIT) can be used to model chafes between these modes. Because FIT is computationally slow, by simulating limited number of points, a curve fitting technique can be used to predict the chafe profile within 0.01% of the simulation values. A polynomial equation is generated to represent the chafe fault profile that can be used in the inversion scheme to predict the nature of the fault. The coaxial cable example is presented, but the same concept can be used in other wire types.

The modified transmission matrix (MTM) and S-parameter methods provide quick yet realistic solutions to transmission modeling. MTM simplifies the transmission line by representing each line section with a single ABCD matrix. Thus, modeling a cascaded transmission line system is easily done by connecting the modulated blocks. The ABCD matrices not only represent the characteristic impedances of transmission lines, they also characterize the reactive components such as capacitors and inductors. These frequency-dependent components are typically not simulated in the time domain methods due to the limited capabilities. Unlike its time domain counterparts which divide wires into numerous small sections or meshes, the performance of the MTM and S-parameter methods does not depend on the resolution of the transmission length. Therefore, the modeling process requires fewer computational resources. The S-parameter method was also used to simulate the reflectometry response in the frequency and time domains in a matrix approach, with accuracy of about 0.3% of measured reflectometry profiles.

These new methods have proved highly useful for simulation and analysis of complex systems. Results can be obtained by using detailed models of the faults and a method to integrate multiple fault models (which can include measured data) in a unified forward model that describes effects of the fault and its surrounding system. Models of shielded cables can be used, where the external environment has little or no impact on the cable, and thus potentially enable the diagnosis of much smaller faults than have previously been detectable. This can lead to accurate identification, location, and diagnosis of faults with high precision, providing real solutions for greater safety and reparability in aerospace wiring systems.

REFERENCES